THE THEORY OF ELLIPTIC INTEGRALS AND THE PROPERTIES OF SURFACES OF THE SECOND ORDER, APPLIED TO THE INVESTIGATION OF THE MOTION OF A BODY ROUND A FIXED POINT

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BY

JAMES BOOTH, LL.D., F.R.S., &c.

CHAPLAIN TO THE MOST HONOURABLE THE MARQUESS OF LANSDOWNE,
AND FORMERLY PRINCIPAL OF BRISTOL COLLEGE.

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1851.

"Quant aux sciences des phénomènes naturels, nous ne doutons point que les surfaces du second degré ne doivent s'y présenter aussi dans un grand nombre de questions, et y jouer un rôle aussi important que celui des sections coniques dans le système planétaire." — CHASLES, *Aperçu Historique*, p. 251.

TO

THE MOST HONOURABLE

THE MARQUIS OF LANSDOWNE, K.G.

LORD PRESIDENT OF THE COUNCIL,

THE FOLLOWING TREATISE

IS RESPECTFULLY INSCRIBED

BY

THE AUTHOR.

PREFACE.

THE investigations given in the following pages were made, the greater portion of them, several years ago. Some of them appeared from time to time in those periodical publications whose pages are open to discussions on subjects of this nature.

In this treatise a complete investigation has been attempted of the laws of the motion of a rigid body round a fixed point, free from the action of accelerating forces, based on the properties of surfaces of the second order, of the curves in which these surfaces intersect, and on the theory of elliptic integrals. The results which have been obtained are exact and not approximate, general and not restricted by any imposed hypothesis.

That the theory of the rotation of a rigid body round a fixed point might be made to rest on the properties of the ellipsoid, was long ago shown by Legendre, and more recently by Poinsot in his brief but elegant tract, the "*Théorie nouvelle de la Rotation des Corps.*" Professor De Morgan very justly observes, in his great work on the Differential and Integral Calculus, "that the long, isolated, and inelegant investigations which usually fill up the chapters of works on dynamics which treat of rotatory motions might be almost entirely avoided, if the student were supposed to have that knowledge of the ellipsoid which he is supposed to have of the ellipse before he reads on the theory of gravitation." The ultimate analysis, however, or the dynamical solution of this problem, must be sought in the evaluation of those mathematical expressions known as elliptic integrals. At this point writers usually have abandoned the subject, or confined themselves to the discussion of particular hypotheses, and the deduction of approximate results.

In connection with this portion of the subject, two curves, which have not, within my knowledge, been noticed by geometers, will claim the reader's attention. I have named them the *spherical parabola*, and the *logarithmic ellipse*. The former may be traced on the surface of a sphere, the latter on a paraboloid of revolution. These curves are of importance. They are the geometrical representatives of elliptic integrals of the first order, and of the logarithmic form of the third order. They complete the geometrical expressions for those integrals. The *four* forms may be represented by four ellipses, one *plane*, two *spherical*, and one *parabolic*. The formulæ of comparison for those integrals given by Legendre and by others follow as simple geometrical conclusions from the properties of these curves. As an example, may be mentioned, Lagrange's scale of modular transformations in the first order, and the linear logarithmic and circular residuals which present themselves in the comparison of the different orders of elliptic integrals.

I have carefully abstained from introducing any methods which, to one moderately versed in the first principles of the integral calculus, might not fairly be assumed as known. There is one exception. In a few cases, where the method was peculiarly applicable, I have ventured to make use of a new kind of coordinates, which were named in a short tract, published some years ago, *tangential coordinates*. The reader may, however, if he chooses, omit those applications, without breaking the continuity of the subject.

I have not been led away by mathematical pedantry to attempt to render this essay purely algebraical, by rejecting geometrical conceptions and the aids thence derived to simplicity and clearness; knowing that, very often, the elegance of the analysis is owing to the distinctness of the graphical conception, and that though the forms of the reasoning may be different, the subject matter is identically the same.

J. B.

May 1*st*, 1851.