ELEMENTS OF THE METHOD OF LEAST SQUARES

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Elements of the Method of Least Squares by Mansfield Merriman

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MANSFIELD MERRIMAN

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METHOD OF LEAST SQUARES.

BY

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PREFACE.

In writing the following pages I have had two objects in view: first, to present the fundamental principles and processes of the Method of Least Squares in so plain a manner, and to illustrate their application by such simple and practical examples, as to render it accessible to Civil Engineers who have not had the benefit of extended mathematical training; and secondly, to give an elementary exposition of the theory which would be adapted to the needs of a large and constantly increasing class of students.

Hence the work falls into two parts, the first practical and the second theoretical, but each illustrating and supplementing the other. The numbering of the articles renders reference from one to the other easy; and the more thorough acquaintance the engineer makes with the second part the better will he adjust his observations, while it is only after much exercise with practical problems that the student can become thoroughly grounded in the theory.

Should the book, then, be taken up by students unfamiliar with the subject, let me suggest to them, that even if their aim be only to acquire a knowledge of its theory, the shortest and best way to do it is to become first familiar with the practical applications of Part I.; this attained, the rest follows naturally and easily.

As I have not written for mathematical experts, they will doubtless find considerable in the book at which to grumble. The idea of mean error does not appear. The term "equations of condition" has been, in accordance with the sensible German practice, divided into "observation equations" and "conditional equations" (*Beobachtungsgleichungen* and *Bedingungsgleichungen*), and each is used in its proper place. GAUSS' development of the law of probability of error has been followed as the best adapted to an elementary presentation, and if this be objected to as defective, I claim at least the credit of knowing and of pointing out (Art. 66) just what and where those defects are.

In preparing these pages I have consulted and freely used all the works upon the subject within my reach. The list of Literature and the historical notice at the end of the book will be of interest and value to all.

If this little elementary work should meet with a favourable reception from the scientific public, it may be followed by another containing extended applications of the method to higher geodetic surveying, and to numerous other problems arising in physical science, which have here been necessarily left unnoticed.

MANSFIELD MERRIMAN.

SHEFFIELD SCIENTIFIC SCHOOL,
NEW HAVEN, CONN., U.S.A., *Feb.* 5, 1876.

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