ELECTRIC WAVES: BEING AN ADAMS PRIZE ESSAY IN THE UNIVERSITY OF CAMBRIDGE

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ELECTRIC WAVES

BEING AN ADAMS PRIZE ESSAY IN THE
UNIVERSITY OF CAMBRIDGE

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PREFACE.

THE following essay was undertaken with the object of discussing the possibility of obtaining directly from Faraday's laws a consistent scheme for the representation of electrical phenomena, and of applying the results to obtain the quantitative relations which exist in certain cases of the propagation of electrical effects.

Maxwell's memoir on "A dynamical theory of the Electromagnetic Field," communicated to the Royal Society in October, 1864, marks a new departure in electrical theory. In it the analytical representation of Faraday's laws is systematically developed and applied, as also the analytical formulation of the electromagnetic theory of light which had already been proposed by Faraday in 1846; but these contributions to electrical theory, though of great importance, are subservient to the main object of the paper, which is to shew that the laws of electrical phenomena obey the same general principle as the laws of mechanics. It has not been sufficiently noticed that Maxwell presented his theory under two forms, in one of which the electrokinetic energy is expressed in terms of currents, and in the other in terms of magnetic force and induction. The first form is the one used throughout the paper, the second form being given without application; and in his Treatise the first form is used in the discussion of the general theory, the second form being given later and only applied to discuss the pressure of radiation and magneto-optic relations. Subsequent writers, FitzGerald, Heaviside, Hertz and others, have taken the second form of the energy function as the starting-point of their investigations.

The fact, that in certain cases the direct application of Faraday's laws gives without ambiguity results different from those which appear to follow from the latter form of Maxwell's theory, led the writer to suspect that there must be some flaw in the process of reasoning by which this form of Maxwell's theory is deduced from Faraday's laws. This suggested the procedure adopted in this essay, to begin by applying Faraday's laws, without the intervention of any dynamical theory, to the different cases which can arise, and to examine whether the results obtained are consistent with observation. A satisfactory scheme having been developed in this way, a short account of Maxwell's procedure is given with the view of discovering the source of the discrepancy, and the result of examination is to shew that the first form in which Maxwell presented his theory is a logical consequence of Faraday's laws while the second form is not. That this has not been noticed earlier is to be explained by the fact that Maxwell did not use the second form of his theory to obtain results whereas subsequent writers have used it in preference to the first form, and that, so far as the applications made by Maxwell are concerned, the same results follow from either form of the theory. That the second form of the theory is easier to work with is obvious, as in this form the Lagrangian function is of the same type as the Lagrangian function of a material medium, thus allowing the argument from analogy to be used, and this explains the preference shewn for this form of the theory.

Having found that the second form of the theory is not logically involved in the first, it becomes necessary to ascertain what assumptions its use has led to, and it appears that many of the received ideas as to the nature of the aether, as for example the doctrine of a fixed aether, have arisen in this way. These assumptions not being directly concerned in the first form of Maxwell's theory, and this form being the one which logically follows from Faraday's laws, the ideas that have arisen from the assumptions cannot be regarded as being required by the facts.

The next step is to examine whether the form of Maxwell's theory thus adopted is consistent with the laws of dynamics, and the logical development of this form of the theory is then resumed at the point where it was left by Maxwell. This latter is perhaps unnecessary in view of the fact that earlier in the essay it is shewn that Faraday's laws are sufficient in themselves for the development of a scheme of representation; but it was thought desirable to add it for the sake of completeness.

The object of the second part of the essay is the application of the general theory to some of the problems that present themselves in connection with the propagation of electrical effects. It appears that there is an essential difference between a simply-connected and a multiply-connected space in respect of the propagation of electrical effects, there being no permanent free oscillations such as would not be dissipated by radiation belonging to an indefinitely extended simply-connected space, while there are such permanent free oscillations associated with each of the conducting circuits that render an indefinitely extended space a multiply-connected region. The explicit recognition of this fact makes it possible to simplify the mathematical theory of electric waves. The complete determination of the circumstances of propagation of waves of the latter class can be reduced to the solution of linear differential equations involving one independent variable: the dependent variable belonging to any conducting path returns to the same value on going once round the path when closed, and the waves circulate without decay. In an open path the dependent variable vanishes at both ends and the energy is dissipated in radiation. The principal application of the theory is to the effects investigated by Hertz in his experiments on electric waves. The development of the analysis gives results which are in close agreement with the observations of Sarasin and de la Rive, who repeated Hertz's experiments under more favourable conditions; it also gives a satisfactory explanation of the discrepancies in some of Hertz's own observations. The investigation thus given for the free periods of resonators is only approximate; the accurate equation for the determination of the periods of a resonator can be easily deduced from the general theory of these waves, but this equation is such that the calculation of the periods from it would involve great labour, and as the error involved in using the approximate theory, in which the distance between successive nodes is approximately half a wave-length, must be extremely small, the gain in theoretical accuracy did not appear to be sufficient compensation for the additional labour.

The essay in its present form was completed at the end of 1900, with the exception of Chapter VIII, the paragraph relating to it in Chapter I, and the Appendices. The paragraphs have also been renumbered throughout and the cross references altered accordingly. Chapter VIII has been added with the view of developing the manner in which the energy of permanent vibrations associated with closed circuits is distributed, and the distinction between them and the waves due to open oscillators. To effect this it was found desirable for the sake of uniform treatment to give a short account of radiation, starting from the first form of Maxwell's theory. Appendices A and B are intended to elucidate the point of view adopted in the first part of the essay. Appendix C supplies the analysis