FIRST PRINCIPLES OF ALGEBRA: ADVANCED COURSE, PP. 277-480

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Trieste

FIRST PRINCIPLES OF ALGEBRA

Advanced Course

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PREFACE

THE Advanced Course of the First Principles of Algebra contains a review of all topics treated in the Elementary Course, together with such additional topics as are required to make it amply sufficient to meet the entrance requirements of colleges or technical schools. Its development is based upon the following important considerations:

1. The pupil has had a one year's course in algebra, involving constant application of its elementary processes to the solution of concrete problems. This has invested the processes themselves with an interest which now makes them a proper object of study for their own sake.

2. The pupil has, moreover, developed in intellectual maturity and is, therefore, able to comprehend processes of reasoning with abstract numbers which were entirely beyond his reach in the first year's course. This is particularly true if, in the meantime, he has learned to reason with the more concrete forms of geometry.

In consequence of these considerations, the treatment throughout is from a more mature point of view than in the Elementary Course. The principles of algebra are stated and proved in logical form, upon the basis of a definite set of axioms.

As in the Elementary Course, the important principles are used at once in the solution of concrete and interesting problems, which, however, are here adapted to the pupil's greater maturity and experience. But relatively greater space and emphasis are given to the manipulation of standard algebraic forms, such as the student is likely to meet in later work in mathematics and physics, and especially such as were too complicated for the Elementary Course.

The division of the First Principles of Algebra into two distinct courses has made it possible to give in the Advanced Course a more thorough treatment of the elements of algebra than could be given if the book were designed for first-year classes. It has thus become possible to lay emphasis upon the pedagogic importance of viewing each subject a second time in a manner more profound than is possible on a first view.

Attention is specifically called to the following points:

The scientific treatment of axioms in Chapter I.

The clear and simple treatment of equivalent equations in Chapter III.

The discussion by formula, as well as by graph, of inconsistent and dependent systems of linear equations, pages 318 to 322.

The unusually complete treatment of factoring and the clear and simple exposition of the general process of finding the Highest Common Factor, in Chapter V.

The careful discrimination in stating and applying the theorems on powers and roots in Chapter VI.

The unique treatment of quadratic equations in Chapter VII, giving a lucid exposition in concrete and graphical form of distinct, coincident, and imaginary roots.

The concise treatment of radical expressions in Chapter X, and especially — an innovation much needed in this connection — the rich collection of problems, in the solution of which radicals are applied.

The authors gratefully acknowledge the receipt of many helpful suggestions from teachers who have used their High School Algebra.

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N. J. LENNES.

CHICAGO AND NEW YORK,
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ADVANCED COURSE

PART THREE

CHAPTER I

FUNDAMENTAL LAWS

1. We have seen in the Elementary Course that **algebra**, like arithmetic, deals with **numbers** and with operations upon numbers. We now proceed to study in greater detail the laws that underlie these operations.

THE AXIOMS OF ADDITION AND SUBTRACTION

2. In performing the elementary operations of algebra we assume at the outset certain simple statements called **axioms**.

Definition. Two number expressions are said to be equal if they represent the same number.

Axiom I. *If equal numbers are added to equal numbers, the sums are equal numbers.*

That is, if $a = b$ and $c = d$, then $a + c = b + d$.

Axiom I implies that *two numbers have one and only one sum.* This fact is often referred to as the **uniqueness of addition**.

3. If $a = c$ and $b = c$ then $a = b$, since the given equations assert that a is the same number as b. Hence the usual statement: *If each of two numbers is equal to the same number, they are equal to each other.*

4. The sum of two numbers, as 6 and 8, may be found by adding 6 to 8 or 8 to 6, in either case obtaining 14 as the result.

This is a particular case of a general law for all numbers of algebra, which we state as

Axiom II. *The sum of two numbers is the same in whatever order they are added.*

This is expressed in symbols by the identity:

$$a + b = b + a. \qquad \text{[See § 75, E. C.*]}$$

Axiom II states what is called the **commutative law of addition**, since it asserts that numbers to be added may be *commuted* or interchanged in order.

Definition. Numbers which are to be added are called **addends.**

5. In adding three numbers such as 5, 6, and 7 we first add two of them and then add the third to this sum. It is immaterial whether we first add 5 and 6 and then add 7 to the sum, or first add 6 and 7 and then add 5 to the sum. This is a particular case of a general law for all numbers of algebra, which we state as

Axiom III. *The sum of three numbers is the same in whatever manner they are grouped.*

In symbols we have $\quad a + b + c = a + (b + c).$

When no symbols of grouping are used, we understand $a + b + c$ to mean that a and b are to be added first and then c is to be added to the sum.

Axiom III states what is called the **associative law of addition,** since it asserts that addends may be *associated* or grouped in any desired manner.

It is to be noted that an equality may be read in either direction. Thus $\quad a + b + c = a + (b + c)$ and $a + (b + c) = a + b + c$ are equivalent statements.

6. If any two numbers, such as 19 and 25, are given, then in arithmetic we can always find a number which added to the smaller gives the larger as a sum. That is, we can subtract the smaller number from the larger.

* E. C. means the Elementary Course.