

TRIGONOMETRY AND DOUBLE ALGEBRA

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649465972

Trigonometry and Double Algebra by Augustus De Morgan

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

AUGUSTUS DE MORGAN

**TRIGONOMETRY
AND
DOUBLE ALGEBRA**

TRIGONOMETRY
AND DOUBLE ALGEBRA

BY

AUGUSTUS DE MORGAN

OF TRINITY COLLEGE, CAMBRIDGE

RECIPIENT OF THE ROYAL ASTRONOMICAL SOCIETY
FELLOW OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY
AND PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

La seule manière de bien traiter les éléments d'une science exacte et rigoureuse, c'est d'y mettre toute la rigueur et l'exhaustivité possible.—D'ALEMBERT.

Tout ce que l'algèbre et la géométrie ont été séparées, leur propre ont été joints et leurs usages réunis; mais lorsque ces deux sciences se sont réunies, elles se sont jointes des forces matérielles, et ont marché ensemble d'un pas rapide vers la perfection.—LAGRANGE.

LONDON:

PRINTED FOR TAYLOR, WALTON, AND MABERLY,

BOOKSELLERS AND FULDESMEN TO UNIVERSITY COLLEGE,

UPPER GOWER STREET AND IVY LANE, PATERNOSTER ROW.

1849.

CAMBRIDGE:
PRINTED BY MORGAN AND PALMER, TRINITY STREET.

PREFACE.

THE work before the reader is entirely new, not being in any sense a second edition of that which I published on the same subject in 1837.

It consists of two books. In the first, I have endeavoured to give the student who has a competent knowledge of arithmetic and algebra—as much for instance as is contained in my works on those subjects, to which reference is made in various places—a view of trigonometry, as a branch of algebra and a constituent part of the foundation of the higher mathematics. In the second, I have given an elementary view of algebra in its purely symbolic character, with the application of that geometrical basis of significance which affords explanation of every symbol.

The term *double algebra* has not yet obtained currency, though that of *triple algebra* has, of late years, been much employed. It means algebra in which

each symbol stands for an object of thought having two distinct and independent qualities: just as the symbol of a straight line, to be perfect, must designate both the length and direction of the line. I have not, after much thought, and some discussion, been able to fix on a better name of sufficient brevity. If, by the application of a somewhat startling adjective to the word *algebra*, any of those who are still bewildered by an art in which *impossible quantities*, or quantities which are not quantities, are made objects of reasoning, should become aware that by slow degrees, and the union of many heads, the art has become a science, and the impossibilities possible, they, at least, will have no objection to the phrase.

A. DE MORGAN.

University College, London.
Feb. 10, 1819.

LIST

OF SOME WRITINGS ON THE SUBJECT OF ALGEBRA,

In which the peculiar Symbols of Algebra are discussed.

- London, 1685, folio. JOHN WALLIS. *A Treatise of Algebra, both historical and practical*. Reprinted in Latin, with additions, in the second volume of *Wallis's Works*, Lond. 1685, folio.
- Naples, 1687, folio. GILES FRANCIS DE GOTTIFRIDEIS. *Logistica Universalis*.
- London, 1758, 4to. FRANCIS MARRERS. *A Dissertation on the use of the Negative Sign in Algebra*.
- London, 1790, 8vo. WILLIAM BRUNN. *The Principles of Algebra*.²
- Cambridge, 1823, 4to. ROBERT WOODHOUSE. *The Principles of Analytical Calculation*.
- Philosophical Transactions for 1805*. M. L'ABBE BUIE. *Mémoire sur les Quantités Imaginaires* (Read June 20, 1805). See also the review of this in Vol. VII. of the *Edinburgh Review*, April—July, 1808 (written by PLATTBY).
- London, 1817, 4to. BENJAMIN GOMPERTZ. *The Principles and Application of Imaginary Quantities, Book I., to which are added some observations on persons*.....
- London, 1818, 4to. BENJAMIN GOMPERTZ. *The Principles and Application of Imaginary Quantities, Book II., derived from a particular case of functional projections*.....
- Paris, 1828, 8vo. (small). G. V. MORSEY. *La vraie théorie des Quantités Algébriques, et des Quantités Préliminaires Imaginaires. Déjà aux amis de l'évidence*.
- Cambridge, 1828, 8vo. JOHN WARREN. *A Treatise on the Geometrical Representation of the Square Roots of Negative Quantities*. *Philosophical Transactions for 1829*. JOHN THOMAS CHALKER. "An attempt to rectify the inaccuracy of some logarithmic formulae." (Read December 18, 1828.)
- Philosophical Transactions for 1829*. JOHN WARREN. "Consideration of the objections raised against the geometrical representation of the square roots of negative quantities." (Read February 19, 1829.) The same volume contains JOHN WARREN. "On the geometrical representation of the powers of quantities, whose indices involve the square roots of negative quantities." (Read June 4, 1829.)
- Cambridge, 1830, 8vo. GEORGE PEACOCK. *A Treatise on Algebra*.
- Cambridge, 1837, 8vo. ANONYMUS [OSBORNE REYNOLDS]. *Strictures on certain parts of 'Peacock's Algebra,' by a Graduate*.

² An opponent not only of imaginary but of negative quantities. Perhaps this work suggested M. BUIE's memoir. I have a letter in my possession from M. BUIE to Mr. VECCHI, dated June 21, 1801, by which it appears that the former was distressed by a gentleman in whose house he was living (at that time, perhaps) to write a private reply to Mr. TREUB'S objections. This letter evidently contains the germ of the views which he afterwards published. See the Annual Report of the Royal Astronomical Society for 1842. According to Dr. PEACOCK, M. BUIE is the first formal maintainer of the geometrical signification of $\sqrt{-1}$.

vi LIST OF SOME WRITINGS ON ALGEBRA.

- Philosophical Transactions* for 1831. DAVIES GILBERT. 'On the nature of negative and of imaginary quantities.' (Read November 18, 1830.)
- London, 1834, 8vo. *Report of the Third Meeting of the British Association for the Advancement of Science*. This volume contains George Peacock's 'Report on certain branches of analysis,' a most valuable historical dissertation on, among other things, the advance of algebra. I cite from it the following works, which I have either not seen, or cannot immediately obtain. Paris, 1806, ARBAUD, *Essai sur la manière de représenter les Quantités Imaginaires dans les constructions géométriques*. Also papers or observations by FRANÇOIS, ARBAUD, SÉVÉTOIS, OUDONNET, in the *Annales des Mathématiques* for 1813 (and I suppose the following year). Also a paper on the arithmetic of impossible quantities, by PRATTAIN, in the *Philosophical Transactions* for 1778; with a Reply, by WOODHOUSE, in the same work for 1802, entitled 'On the necessary truth of certain conclusions obtained by aid of imaginary expressions.'
- London, 1836, 8vo. ANONYMOUS (FRANCIS PEACOCK). *A Synopsis of a Course of Lectures upon Trigonometry, and the Application of Algebra to Geometry*.
- London, 1837, 8vo. A. DE MORGAN. *Elements of Algebra*. 2nd edition.
- London, 1837, 8vo. A. DE MORGAN. *Elements of Trigonometry and Trigonometrical Analysis, preliminary to the Differential Calculus*, . . .
- Edinburgh Philosophical Transactions*, Vol. XIV, Part 1. DUCAN [FRANCIS] GREGORY. 'On the real nature of Symbolical Algebra.' (Read May 7, 1838.)
- Lectures' Diary*. London, 1839, 8vo. (small). THOMAS WHITE. 'On the algebraical expansion of quantity, . . . and on the symbol $\sqrt{-1}$, which is usually considered* to denote impossible or imaginary quantity,' (at page 39).
- Cambridge *Philosophical Transactions*, Vol. VII, Part 2. A. DE MORGAN. 'On the Foundation of Algebra.' (Read Dec. 9, 1839.)
- Cambridge *Philosophical Transactions*, Vol. VII, Part 3. A. DE MORGAN. 'On the Foundation of Algebra, No. II.' (Read Nov. 29, 1841.)
- Paris, 1841, 8vo. M. F. VALLÉ. *Études Philosophiques sur la science du calcul*. Première Partie. No more yet published.
- Cambridge *Philosophical Transactions*, Vol. VIII, Part 2. A. DE MORGAN. 'On the Foundation of Algebra, No. III.' (Read Nov. 27, 1843.)
- Cambridge, 1842 & 1845, 8vo. GEORGE PEACOCK. *A Treatise on Algebra*. Vol. I.—Arithmetical Algebra. Vol. II.—Symbolical Algebra and its applications to the geometry of position.
- London, 1845, 12mo. MARTIN OLM [translated by ALEXANDER JOHN HILL]. *The Spirit of Mathematical Analysis, and its relation to a logical system*.

* The author supposes it to be indeterminate, because it can be expanded by help of a divergent series. The paper is marked 'received April 1846.'

TABLE OF CONTENTS.

♦. THE REFERENCES ARE TO THE PAGES OF THE WORK.

BOOK I.—TRIGONOMETRY.

CHAPTER I.

Preliminary Notions.

Definition of trigonometry, 1; undulating magnitude, 1; periodic magnitude, 2; suggested by angular magnitude, 2; *gradual* measurement of angle, 3; factors of 360, 3; circumference of circle, 4; π , 4; multiplication and division by π , 5; arc \div radius, 5; arcual measurement of angle, 5, 6; gradual and arcual comparisons, 6; gradual measurement of arc, 6.

CHAPTER II.

On the Trigonometrical Functions, and on Formulae of One Angle.

Axis, origin, projections, co-ordinates, abscissa, ordinate, 7; r , θ , x , y , 7; sign of r , x , y , 8; four quarters and their signs, 8; θ and $2m\pi + \theta$, 9; sine, cosine, tangent, cotangent, secant, cosecant, versed sine, covered sine, 9; complement, supplement, opponent, completion, 10; trigonometrical functions as abstract numbers and multipliers, 10; curve of sines, &c., 11; fundamental equations, 11, 12; limits of value, 12; signs, 12, 13; negative sign of r , 13; initial and terminal values, 13; cosine even, sine odd, 13; tangent odd, 14; $\frac{1}{2}\pi \pm \theta$ and its rules, 14, 15; double value of functions, 16; 15° , 18° , 30° , 45° , 60° , 72° , 75° , 16, 17; $\sin \theta \div \theta$, $(1 - \cos \theta) \div \theta$, $\tan \theta \div \theta$, 17, 18; θ and $1 - \frac{1}{2}\theta$, 18; older system of definitions, 18, 19; area of circle and sector, 20.