TRIGONOMETRY AND DOUBLE ALGEBRA

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AND DOUBLE ALGEBRA

BY

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La seule manière de bien traiter les élémens d'une science exacte est rigoureuse, c'est d'y mettre toute la rigueur et l'exactitude possible.—D'ALEMBERT.

Tant que l'algèbre et la géométrie ont été séparées, leur progrès ont été lents et leurs usages bornés; mais lorsque ces deux sciences se sont réunies, elles se sont prêtées des forces mutuelles, et ont marché ensemble d'un pas rapide vers la perfection.—LAGRANGE.

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PRINTED BY METCALFE AND PALMER, TRINITY STREET.

PREFACE.

THE work before the reader is entirely new, not being in any sense a second edition of that which I published on the same subject in 1837.

It consists of two books. In the first, I have endeavoured to give the student who has a competent knowledge of arithmetic and algebra—as much for instance as is contained in my works on those subjects, to which reference is made in various places—a view of trigonometry, as a branch of algebra and a constituent part of the foundation of the higher mathematics. In the second, I have given an elementary view of algebra in its purely symbolic character, with the application of that geometrical basis of significance which affords explanation of every symbol.

The term *double algebra* has not yet obtained currency, though that of *triple algebra* has, of late years, been much employed. It means algebra in which

each symbol stands for an object of thought having two distinct and independent qualities: just as the symbol of a straight line, to be perfect, must designate both the length and direction of the line. I have not, after much thought, and some discussion, been able to fix on a better name of sufficient brevity. If, by the application of a somewhat startling adjective to the word *algebra*, any of those who are still bewildered by an art in which *impossible quantities*, or quantities which are not quantities, are made objects of reasoning, should become aware that by slow degrees, and the union of many heads, the art has become a science, and the impossibilities possible, they, at least, will have no objection to the phrase.

A. DE MORGAN.

University College, London,
Feb. 10, 1849.

LIST

OF SOME WRITINGS ON THE SUBJECT OF ALGEBRA,

In which the peculiar Symbols of Algebra are discussed.

London, 1685, folio. JOHN WALLIS. *A Treatise of Algebra, both historical and practical.* Reprinted in Latin, with additions, in the second volume of *Wallis's Works*, Lond. 1693, folio.

Naples, 1687, folio. GILES FRANCIS DE GOTTIGNIES. *Logistica Universalis.*

London, 1758, 4to. FRANCIS MASERES. *A Dissertation on the use of the Negative Sign in Algebra.*

London, 1796, 8vo. WILLIAM FREND. *The Principles of Algebra.*[*]

Cambridge, 1803, 4to. ROBERT WOODHOUSE. *The Principles of Analytical Calculation.*

Philosophical Transactions for 1806. M. L'ABBÉ BUÉE. *Mémoire sur les Quantités Imaginaires* (Read June 20, 1805). See also the review of this in Vol. XII. of the *Edinburgh Review*, April —July, 1808 (written by Playfair).

London, 1817, 4to. BENJAMIN GOMPERTZ. *The Principles and Application of Imaginary Quantities*, Book I., *to which are added some observations on paisms.....*

London, 1818, 4to. BENJAMIN GOMPERTZ. *The Principles and Application of Imaginary Quantities*, Book II., *derived from a particular case of functional projections....*

Paris, 1828, 8vo. (small). C. V. MOURNEY. *La vraie théorie des Quantités Négatives, et des Quantités Prétendues Imaginaires. Dédié aux amis de l'évidence.*

Cambridge, 1828, 8vo. JOHN WARREN. *A Treatise on the Geometrical Representation of the Square Roots of Negative Quantities.*

Philosophical Transactions for 1829. JOHN THOMAS GRAVES. 'An attempt to rectify the inaccuracy of some logarithmic formulæ.' (Read December 18, 1828.)

Philosophical Transactions for 1829. JOHN WARREN. 'Consideration of the objections raised against the geometrical representation of the square roots of negative quantities. (Read February 19, 1829.) The same volume contains JOHN WARREN. 'On the geometrical representation of the powers of quantities, whose indices involve the square roots of negative quantities.' (Read June 4, 1829.)

Cambridge, 1830, 8vo. GEORGE PEACOCK. *A Treatise on Algebra.*

Cambridge, 1837, 8vo. Anonymous [OSBORNE REYNOLDS]. *Strictures on certain parts of 'Peacock's Algebra,'* by a Graduate.

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[*] An opponent not only of imaginary but of negative quantities. Perhaps this work suggested M. Buée's memoir. I have a letter in my possession from M. Buée to Mr. Frend, dated June 21, 1801, by which it appears that the former was desired by a gentleman in whose house he was living (as tutor, perhaps) to write a private reply to Mr. Frend's objections. This letter evidently contains the germs of the views which he afterwards published. See the Annual Report of the Royal Astronomical Society for 1842. According to Dr. Peacock, M. Buée is the first formal maintainer of the geometrical signification of $\sqrt{-1}$.

Philosophical Transactions for 1831. DAVIES GILBERT. 'On the nature of negative and of imaginary quantities.' (Read November 18, 1830.)

London, 1834, 8vo. *Report of the Third Meeting of the British Association for the Advancement of Science.* This volume contains George Peacock 'Report on certain branches of analysis,' a most valuable historical discussion on, among other things, the advance of algebra. I cite from it the following works, which I have either not seen, or cannot immediately obtain. Paris, 1806, ARGAND, *Essai sur la manière de représenter les Quantités Imaginaires dans les constructions géométriques.* Also papers or observations by FRANÇOIS, ARGAND, SERVOIS, GERGONNE, in the *Annales des Mathématiques* for 1813 (and I suppose the following year). Also a paper on the arithmetic of impossible quantities, by PLAYFAIR, in the *Philosophical Transactions for* 1778; with a Reply, by WOODHOUSE, in the same work for 1802, entitled 'On the necessary truth of certain conclusions obtained by aid of imaginary expressions.'

London, 1836, 8vo. ANONYMOUS [GEORGE PEACOCK]. *A Syllabus of a Course of Lectures upon Trigonometry, and the Application of Algebra to Geometry.*

London, 1837, 8vo. A. DE MORGAN. *Elements of Algebra.* 2nd edition.

London, 1837, 8vo. A. DE MORGAN. *Elements of Trigonometry and Trigonometrical Analysis, preliminary to the Differential Calculus,....*

Edinburgh Philosophical Transactions, Vol. XIV. Part 1. D[UNCAN] F[ORBES] GREGORY. 'On the real nature of Symbolical Algebra.' (Read May 7, 1838).

Ladies' Diary. London, 1839, 8vo. (small). THOMAS WHITE. 'On the algebraical expression of quantity,... and on the symbol $\sqrt{-1}$, which is usually considered* to denote impossible or imaginary quantity,' (at page 39).

Cambridge Philosophical Transactions, Vol. VII. Part 2. A. DE MORGAN. 'On the Foundation of Algebra. (Read Dec. 9, 1839).

Cambridge Philosophical Transactions, Vol. VII. Part 3. A. DE MORGAN, 'On the Foundation of Algebra, No. II. (Read Nov. 29, 1841).

Paris, 1841, 8vo. M. F. VALLÈS. *Etudes Philosophiques sur la science du calcul.* Première Partie. No more yet published.

Cambridge Philosophical Transactions, Vol. VIII. Part 2. A. DE MORGAN. 'On the Foundation of Algebra, No. III. (Read Nov. 27, 1843).

Cambridge, 1842 & 1845, 8vo. GEORGE PEACOCK. *A Treatise on Algebra.* Vol. I.—Arithmetical Algebra. Vol. II.—Symbolical Algebra and its applications to the geometry of position.

London, 1843, 12mo. MARTIN OHM [translated by ALEXANDER JOHN ELLIS]. *The Spirit of Mathematical Analysis, and its relation to a logical system.*

* The author supposes it to be indeterminate, because it can be expanded by help of a divergent series. The paper is marked 'received April 1816.'

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*** THE REFERENCES ARE TO THE PAGES OF THE WORK.