THE ELEMENTS OF ARITHMETIC

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649469970

The Elements of Arithmetic by Augustus De Morgan

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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AUGUSTUS DE MORGAN

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BY

AUGUSTUS DE MORGAN,

OF TRINITY COLLEGE, CAMBRIDGE; ONE OF THE VICE-PRESIDENTS OF THE ROYAL ASTRONOMICAL SOCIETY; FELLOW OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY; AND PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON.

FOURTH EDITION.

"Ce n'est point par la routine qu'on s'instruit, c'est par sa propre réflexion: et il est essentiel de contracter l'habitude de se rendre raison de ce qu'on fait: cette habitude s'acquiert plus facilement qu'on ne pense: et une fois acquise, elle ne se perd plus."—CONDILLAC.

LONDON:

PRINTED FOR TAYLOR AND WALTON,

BOOKSELLERS AND PUBLISHERS TO UNIVERSITY COLLEGE,

28 UPPER GOWER STREET.

M.DCCC.XL.

LONDON:
PRINTED BY MOYES AND BARCLAY,
Castle Street, Leicester Square.

PREFACE

TO

THE SECOND EDITION.

I HAVE added several new chapters in this Edition, particularly on the Square Root, Proportion, and Permutations and Combinations, with many occasional articles, principally intended to give such ideas on the subject of Algebra as a young arithmetical student may be able, with a little assistance, to comprehend. I have also added six or seven Examples to each Rule, accompanied by the answers. These would be enough for any single pupil, but may not be considered sufficient for a school. To obviate this objection, I proceed to collect some expeditious modes of forming questions, of which the answers shall be readily known. I am aware that the publication of these methods, in a preface equally open to the master and the learner, is something like calling the enemy to council; nevertheless, as the following abbreviations all contain some mathematical principle, and as some facility of computation will be necessary even to make use of them, the master may depend upon it that a pupil who discovers and applies the way to make the answer to any one rule, is fit to pass on to the next.

Addition.—Let a series of numbers be taken, each of which is the *complement* to 10^n of the preceding. Strike out one or more, and arrange the rest miscellaneously. It will be evident how to ascertain whether these have been added up correctly. Many arrangements may be made for recollecting which numbers were struck out: for example, their complements may be made to begin with a given figure, and to be the only ones which begin with that figure.

Another method, preferable perhaps to the former, is the following: Let any series of numbers be taken, such as $a, b, c,$ &c. each of which exceeds the following; let the master form $a-b$, $b-c$, $c-d$, &c. and give the results to the pupil to add together, annexing to them the last number which he used. The answer will be a, which number cannot possibly be recovered by the pupil from the *data*, except by the very operation which he is required to perform. The continued subtractions may be done by one pupil, and the addition made by another; and thus the process may afford examples in the first two rules.

Subtraction.—In addition to the method just explained, the following may be used: Instead of giving one number to be subtracted from another *once* only, let it be required to subtract the first time after time from the second, until it can no longer be subtracted, as in the examples of article 46. This being, in point of fact, a question of division, may be proved by casting out the nines, and this after any number of steps, using the number of subtractions performed as the quotient. Or questions might be formed thus: Subtract 1259 from 12590, until this can no longer be done; or, multiplication by 25, 5, or 9, being very expeditiously done, the minuend might be $25x$, $5x$, or $9x$, and the subtrahend x.

Multiplication and Division.—For these a table of squares and cubes is amply sufficient. The most useful of the kind is "Barlow's Tables," which gives at one glance the square and cube, square and cube roots, factors, and reciprocal of any number under 10,000. From such a table, many thousands of examples in multiplication and division may be derived immediately, with the answers, and many hundreds of thousands more may be obtained from the formula,

$$ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

where a and b are both even, or both odd.

Greatest Common Measure, and Least Common Multiple.—In "Barlow's Tables" is given a list of all prime numbers under 100,000. The multiplication of any two of these by the same number, will give a question in the first rule, with its answer. For the second rule, take any low prime numbers a, b, c, d, &c. and multiply one or more of them by each of the low prime numbers e, f, g, &c. Then will $a\,b\,c\,d$, &c. $\times$ $e\,f\,g$, &c. be the least common multiple of the products above mentioned. All that has been said on the first four rules, applies equally to Common and Decimal Fractions. In the former case, with a table of squares, the formula

$$\frac{a}{a+b} + \frac{b}{a-b} = \frac{a^2+b^2}{a^2-b^2}$$

will be convenient.

What has been said on Addition and Subtraction may also be applied when the quantities are compound, particularly where the examples are sums of money. The verification will be rendered more easy in the latter case, if the rule or table in article 221 be used by the master. A Ready Reckoner, or Table of Interest, will furnish examples in multiplication or division, in a manner too obvious to require any further notice.

In multiplication, different sums, making together £10 or £100, &c. may be given to two pupils to be multiplied by the same number. One result may be made to verify the other, and the answer, in this and all other cases,* should be recorded for future use.

Since the publication of the First Edition of this work, though its sale has sufficiently convinced me that there exists a disposition to introduce the *principles* of arithmetic into schools, as well as the *practice*, I have often heard it remarked that it was a hard book for children. I never dared to suppose it would be otherwise. All who have been engaged in the education of youth are aware that it is a hard thing to make them think; so hard, indeed, that masters had, within the last few years, almost universally abandoned

* The greatest difficulty which an arithmetical master finds, is that of procuring a sufficient number of examples. If he is at all acquainted with algebra, he will be able to propose to two pupils, questions of which the answers shall be simple, though not (to the pupil) obvious, verifications of one another. If in a school there were established a common book, in which the pupil who first succeeded in solving a question should have the privilege of entering the answer, with his name; besides the emulation thereby excited, a collection of examples would be obtained for future use, which would entirely do away with all anxiety on this subject.

the attempt, and taught them rules instead of principles; by authority instead of demonstration. This system is now passing away, and many preceptors may be found who are of opinion that, whatever may be the additional trouble to themselves, their pupils should always be induced to reflect upon, and know the reason of, what they are doing. Such I would advise not to be discouraged by the failure of a first attempt to make the learner understand the principle of a rule. It is no exaggeration to say, that under the present system, five years of a boy's life are partially spent in merely learning the *rules* contained in this treatise, and those, for the most part, in so imperfect a way, that he is not fit to encounter any question unless he sees the head of the book under which it falls. On a very moderate computation of the time thus bestowed, the pupil would be in no respect worse off, though he spent five hours on every page of this work. The method of proceeding which I should recommend, would be as follows: Let the pupils be taught in classes, the master explaining the article as it stands in the work. Let the former then try the demonstration on some other numbers proposed by the master, which should be as simple as possible. The very words of the book may be used, the figures being changed, and it will rarely be found that a learner is capable of making the proper alterations, without understanding the reasoning. The experience of the master will suggest to him various methods of trying this point. When the principle has been thus discussed, let the rule be distinctly stated by the master or some of the more intelligent of the pupils, and let