A HISTORICAL SURVEY OF ALGEBRAIC METHODS OF APPROXIMATING THE ROOTS OF NUMERICAL HIGHER EQUATIONS UP TO THE YEAR 1819

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649342969

A Historical Survey of Algebraic Methods of Approximating the Roots of Numerical Higher Equations up to the Year 1819 by Martin Andrew Nordgaard

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

MARTIN ANDREW NORDGAARD

A HISTORICAL SURVEY OF ALGEBRAIC METHODS OF APPROXIMATING THE ROOTS OF NUMERICAL HIGHER EQUATIONS UP TO THE YEAR 1819

Trieste

A Historical Survey of Algebraic Methods of Approximating the Roots of Numerical Higher Equations up to the Year 1819

By

Martin Andrew Nordgaard, Ph.D.

Assistant Professor of Mathematics,
Grinnell College, Grinnell, Iowa

Teachers College, Columbia University
Contributions to Education, No. 123

Published by
Teachers College, Columbia University
New York City
1922

ACKNOWLEDGMENT

The author is deeply appreciative of the services of Professor David Eugene Smith, of Teachers College, Columbia University, whose inspiration and guidance have been a great help in carrying on this research.

CONTENTS

I

INTRODUCTION

The feature of mathematics which we may call scientific approximation did not receive the recognition among the ancient Greek geometers that has been accorded it by mathematicians of modern times. Since their work in pure mathematics was based on geometry, the Greek scholars evaluated irrational roots as readily as rational roots by the construction of lines. But when it came to applied mathematics, numerical approximation was often not only desirable but absolutely necessary; and so we have some remarkably close approximations by Archimedes (*c.* 225 B. C.) in his work in mechanics, by Heron of Alexandria (*c.* 200 A. D.) in his work as surveyor, and by Theon of Alexandria (*c.* 375 A. D.) in his astronomical computations. Archimedes even used numerical approximations in the domain of pure geometry in his *Measurement of the Circle*. On the whole, however, the Greek mind did not take to this form of mathematical activity.

The Hindus and the Arabs used approximation methods more extensively than the Greeks, and notable work was done by Brahmagupta (*c.* 510 A. D.), Bhāskara (*c.* 1150 A. D.), and an unknown Arab computer mentioned by Chelebī. However, it was during the Renaissance, when the newly discovered general formula for solving the cubic was found to be inoperative for the "irreducible case," that approximation methods became a vital problem, and for two centuries the keenest mathematical minds worked to find smooth, effective methods for approximating cube and higher roots.

The purpose of this research is to trace the history of the different methods of approximating roots of numerical higher equations that were used up to 1819, the date of the publication of Horner's method; we purpose to trace their early beginnings in finding the roots of numbers and in solving incomplete equations, and to watch their growth into systematically developed general methods for solving complete equations of any degree. Pure trial and error methods, like some of the methods of the Egyptians and, in modern times, that of Junge, will not be taken up, nor shall we discuss such ephemeral methods as those of Fontaine, Collins, and some of

DeLagny's. We shall, moreover, limit the investigation to algebraic methods in algebraic equations of one unknown, and only incidentally shall we touch upon transcendental equations and geometric and trigonometic methods.

In searching for literature on this subject the author found only two articles, outside of the encyclopedias, in the nature of surveys: one by Augustus De Morgan in the *Companion to the British Almanac* (1839), entitled "Notices of the Progress of the Problem of Evolution" and dealing with the extension of the Hindu method of evolution to the solution of numerical equations by Vieta and Horner; and a more extended discussion by F. Cajori in an article entitled "A History of the Arithmetical Methods of Approximation to the Roots of Numerical Equations of one Unknown Quantity," in the *Colorado College Publication* (Colorado Springs, Colorado) for 1910. In the present survey we purpose to study the origin and growth of the various methods by bringing out the illustrations and the details of explanation as far as possible from the original sources.

The writer has had unusual opportunities for studying such source materials in four notable libraries,—the Columbia University Library, the Teachers College Library, the New York City Public Library, and the private library of Professor David Eugene Smith, with its unique collection of editions of the Renaissance and early post-Renaissance periods. By this means he has had access to the early editions and standard translations of Euclid, Archimedes, Heron, Theon, Brahmagupta, Mahāvīrā, Bhāskara, Omar Khayyam, as well as the latest brochures and translations of Chinese and Japanese writers on mathematics. He has also been able to consult such works as those of Leonardo of Pisa, Chuquet, Pacioli, Bombelli, Cardan, Stevin, Pitiscus, Vieta, Oughtred, Hume, Hérigone, Wallis, Newton, Raphson, Colson, Rolle, DeLagny, Halley, Taylor, Waring, Euler, Lagrange, and many others who wrote upon the subject previous to Horner.

II

PRE-ALGORITHMIC METHODS

1. EGYPTIAN AND BABYLONIAN WORK IN FINDING ROOTS

We formerly thought that the root concept originated among the Greeks, but recent discoveries show us that the finding of roots of numbers and the solution of quadratic equations, and even the approximation methods employed in this work, go as far back as the early Egyptian and Babylonian civilizations.

On the Senkereh tablets (*c.* 2000 B. C.) are tables of squares and cubes, which shows that the Babylonians had at least an indirect notion of square and cube roots.[1] Three quadratic equations are known to have been studied in this period. The first one was made known when Griffith (1897) published the mathematical papyrus found by Petrie in Kahun.[2] It deals with areas and requires the solution of the equations $xy = 12$ and $x : y = 1 : \frac{3}{4}$, stated, of course, in rhetorical form. In 1900 Schach discovered in a Berlin papyrus a second problem in quadratics, requiring the solution of the equations $x^2 + y^2 = 100$ and $x : y = 1 : \frac{3}{4}$. The third equation was found in the Kahun papyrus by Schach in 1903. It requires the solution of the equations $x^2 + y^2 = 400$ and $x : y = 2 : 1\frac{1}{2}$. The ancient mathematician solved it by letting $x = 2$, $y = 1\frac{1}{2}$; this gives $x^2 + y^2 = 6\frac{1}{4}$; since $\sqrt{6\frac{1}{4}} = 2\frac{1}{2}$ and $2\frac{1}{2} = \frac{1}{8}$ of 20, he found that $x = 2 \cdot 8 = 16$ and $y = 1\frac{1}{2} \cdot 8 = 12$.

The same method, commonly called the method of False Position, is also used in solving the other two equations.

No irrational roots occur in these equations. But there are indications[3] that the Egyptians had a definite way of approaching the square root of non-square numbers. However, they seem to have been unaware of their irrational quality.

2. EARLY CONCEPTS OF HIGHER ROOTS AND IRRATIONALS AMONG THE GREEKS

That some roots are irrational was first recognized by the Greeks. From measuring the areas of commensurable squares and rectangles

[1] M. Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. I (hereafter designated Cantor, I), Leipzig, 1907, p. 28.

[2] M. Simon, *Geschichte der Mathematik im Altertum*, Berlin, 1909, pp. 41–42.

[3] Simon, *Geschichte*, pp. 43–53.