A TREATISE ON ATTRACTIONS, LAPLACE'S FUNCTIONS, AND THE FIGURE OF THE EARTH

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A Treatise on Attractions, Laplace's Functions, and the Figure of the Earth by John H. Pratt

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FIGURE OF THE EARTH.

BY

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PREFACE TO THE THIRD EDITION.

THIS edition is a complete revise of the former ones; improvements are introduced in many places, especially in the Chapters on the Figure of the Earth; and new problems are given either as illustrations of the principles developed in the work, or as bearing upon its main design. An inspection of the Table of Contents will give a comprehensive view of the various matters here taken up.

My chief design in this treatise has been to give an answer to the question, Has the Earth acquired its present Form from being originally in a fluid state? The problem is one of great interest, and involves many investigations of high importance.

Taking the Law of Universal Gravitation as the basis, in the first part of the treatise, I calculate the resultant force exerted on a point by an assemblage of particles endowed with this attracting power, and held together in the form of a sphere, homogeneous or heterogeneous, next of a spheroid, then of an irregular mass consisting of layers nearly spherical, thus approximating more and more to the case of the Earth. This investigation gives me the opportunity of introducing the remarkable analysis of Laplace, which I have endeavoured to put in a clear light, and to free from objections which have been urged against it. The first part of the treatise is closed

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with a Chapter in which is calculated the local effect on the direction of gravity caused by irregular masses at the surface of the Earth, such as exist in table lands, vast mountain regions like the Himmalayas, and hollows filled by the ocean which is of less density than rock; and also wide-spread but slight deficiencies or excesses of matter in the crust below. All these are of importance in the problem which it is my ulterior design to solve, as they furnish the means of explaining anomalies which would otherwise be unaccountable.

The second part of the treatise is occupied in calculating the Figure of the Earth, first upon the hypothesis of its being a fluid mass, and then on geodetical principles. It is shown that the mass now consists of nearly spherical strata, whatever its former history may have been; that, on the fluid hypothesis, the form of its surface and of all its internal layers must be oblate spheroids; and that the plumb-line must be everywhere a normal to its surface. The intimate connexion between the form of the surface and the internal arrangement of the mass is shown by demonstrating the converse of the above; viz. that if the form of the surface be a spheroid of equilibrium, the earth's mass must necessarily be arranged according to the fluid law, whether the mass is or has been fluid, in part or in whole, or not. This is contrary to the belief which some others have entertained. Four tests are next applied to the fluid theory of the earth; by determining what is (1) the law of gravity on the surface which it leads to, (2) the amount of perturbation in the moon's motion in latitude, (3) the amount of precession, and (4) the value of the earth's ellipticity; and by comparing these results with those of observation and experiment. Remarks on the thickness of the earth's crust conclude the first Chapter. In the second the figure is determined geodetically.

The method of Bessel at present in use for this purpose is shown to be erroneous in one particular, and is corrected so as to bring into the calculation the effect of Local Attraction. The degree of uncertainty which that disturbing element brings into the calculation of the figure of the earth is pointed out; and it is shown how, with great probability of a correct result, the ambiguity may be removed by a comparison of the three long arcs, the Anglo-Gallic, the Russian, and the Indian. I believe this is the first time that the mean figure has been calculated, the disturbing effect of local attraction being brought into the calculation throughout. After some propositions on the sea-level, on mapping countries, and on differences of local attraction in the stations of the Indian Arc, the volume is closed by a summary of the argument regarding the hypothesis of the original fluidity of the earth, an hypothesis which I consider to be established beyond doubt.

J. H. PRATT.

CALCUTTA, 1865.

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