

**AN ELEMENTARY  
TREATISE OF  
SPHERICAL GEOMETRY  
AND TRIGONOMETRY**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9781760571962

An Elementary Treatise of Spherical Geometry and Trigonometry by Anthony D. Stanley

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**ANTHONY D. STANLEY**

**AN ELEMENTARY  
TREATISE OF  
SPHERICAL GEOMETRY  
AND TRIGONOMETRY**



AN  
ELEMENTARY TREATISE  
OF  
SPHERICAL GEOMETRY  
AND  
TRIGONOMETRY.

BY  
ANTHONY D. STANLEY, A.M.,  
PROFESSOR OF MATHEMATICS IN YALE COLLEGE.

REVISED EDITION

NEW HAVEN.  
DURRIE AND PECK.  
1854.

EducT 148.54.811

✓

HARVARD COLLEGE LIBRARY  
GIFT OF THE  
GRADUATE SCHOOL OF EDUCATION  
Dec. 16, 1926

---

Entered according to Act of Congress, in the year 1848,  
By ANTHONY D. STANLEY,  
In the Clerk's Office of the District Court of Connecticut.

---

## CONTENTS.

### SPHERICAL GEOMETRY.

	PAGE
Definitions - - - - -	7
Straight Line and Sphere - - - - -	9
Plane and Sphere - - - - -	11
Two Spheres - - - - -	13
Poles of Spherical Circles - - - - -	17
Spherical Angles - - - - -	20
Spherical Triangles - - - - -	22
Comparison of Triangles - - - - -	29
Comparison of Right-angled and Quadrantal Triangles - - - - -	39
Spherical Surfaces - - - - -	46

### SPHERICAL TRIGONOMETRY.

Bi-quadrantal Triangles - - - - -	55
Right-angled Triangles - - - - -	56
Napier's Rules of the Circular Parts - - - - -	62
Oblique-angled Triangles - - - - -	69
Bowditch's Rules for Oblique-angled Triangles - - - - -	88
<i>Subject treated algebraically. (Art. 26)</i> - - - - -	91
Trigonometrical formulæ often used - - - - -	92
Fundamental theorem investigated - - - - -	93
Other theorems deduced from this - - - - -	97
Formulæ prepared for use in Logarithmic Calculations - - - - -	101
Napier's Analogies - - - - -	105
Limitations of value to which the parts of triangles are subject - - - - -	106
Select Formulæ for the Six Cases in the Resolution of Triangles - - - - -	107





## SPHERICAL GEOMETRY.

---

### DEFINITIONS.

1. A *sphere* is a solid such that all points in its surface are equidistant from a certain point within called the *center*.

2. A *radius* of a sphere is any straight line drawn from the center to the surface.

All radii of a sphere are equal.

3. A sphere may be described by the revolution of a semicircle about its diameter, the middle of the diameter being the center, and half the diameter a radius of the sphere.

4. A *diameter* of a sphere is any straight line passing through the center and terminating each way in the surface.

All diameters of a sphere are equal, each of them consisting of two radii.

5. The *axis* of a sphere is a diameter about which the sphere is supposed to have been described by the revolution of a semicircle.

6. Every intersection of a plane with a sphere is a *circle*, as will be seen from the demonstration of Prop. VI.

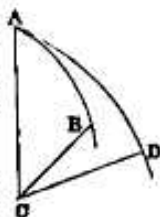
7. The intersection of a sphere with a plane passing through the center is called a *great circle*, and its intersection with any other plane, a *small circle*.

8. The *axis* of a *circle* of a sphere, is that diameter of the sphere which is perpendicular to the circle.

The extremities of the axis are called the *poles* of the circle.

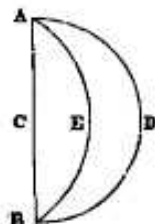
9. The angle made by the arcs of two great circles is called a *spherical angle*, and is to be regarded as the same with the angle between the *planes* of the circles.

Thus,  $BAD$  is a spherical angle, having for its substitute the angle between the planes  $ACB$  and  $ACD$ , supposing  $C$  to be the center of the sphere.



10. A *spherical lune* is a part of the surface of a sphere included between two great semicircles having a common diameter, as  $ADBE$ .

A *spherical ungula* or *wedge* is a part of a sphere, bounded by a lune and the two great semicircles which include the lune, as  $CADBE$ .



11. A *spherical triangle* is a part of the surface of a sphere, included between the arcs of three great circles.

The arcs are called *sides* of the triangle.

12. Spherical triangles are distinguished as *right-angled*, *isosceles*, *equilateral*, &c., in the same way as plane triangles.

A *quadrantal* triangle is that of which one side is a quadrant.

13. A *spherical polygon* is a portion of the surface of a sphere, bounded by several arcs of great circles; which arcs are called *sides* of the polygon.

14. Each side of a triangle or a polygon must be understood to be *less* than a *semicircumference* of a great circle, unless the contrary is stated.

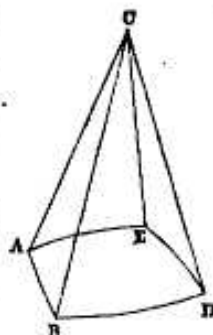
15. A *spherical pyramid* is a part of the sphere, contained by the planes of a solid angle whose vertex is the center, and the spherical polygon included by these planes; as ABCDE.

The polygon is called the *base* of the pyramid.

When the base is a spherical triangle, the pyramid is called *triangular*.

16. A line or plane is said to *touch* or be *tangent* to a sphere, when it meets the surface of the sphere in one point only.

And two spheres are said to *touch* each other, when they meet and do not intersect.



## PROP. I.

*If a perpendicular drawn from the center of a sphere to any straight line be equal to the radius of the sphere, this line touches the sphere at the foot of the perpendicular.*

For since the perpendicular is equal to the radius, the foot of the perpendicular is in the surface of the sphere; the line therefore *meets the surface* at the foot of the perpendicular: and every other point in the line is *without the surface*, being further from the center of