

**THE FOUNDATIONS OF THE  
EUCLIDIAN GEOMETRY AS  
VIEWED FROM THE STANDPOINT  
OF KINEMATICS. DISSERTATION**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649473960

The Foundations of the Euclidian Geometry as Viewed from the Standpoint of Kinematics.  
Dissertation by Israel Euclid Rabinovitch

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.  
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

[www.triestepublishing.com](http://www.triestepublishing.com)

**ISRAEL EUCLID RABINOVITCH**

**THE FOUNDATIONS OF THE  
EUCLIDIAN GEOMETRY AS  
VIEWED FROM THE STANDPOINT  
OF KINEMATICS. DISSERTATION**



THE FOUNDATIONS  
OF THE  
EUCLIDIAN GEOMETRY  
AS VIEWED FROM THE  
STANDPOINT OF KINEMATICS

126148

DISSERTATION  
SUBMITTED TO THE  
BOARD OF UNIVERSITY STUDIES  
OF THE  
JOHNS HOPKINS UNIVERSITY  
IN CONFORMITY WITH THE REQUIREMENTS FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

BY

ISRAEL EUCLID RABINOVITCH

APRIL 29, 1901

NEW YORK:  
PUBLISHED BY THE AUTHOR  
1903

**Dedicated**

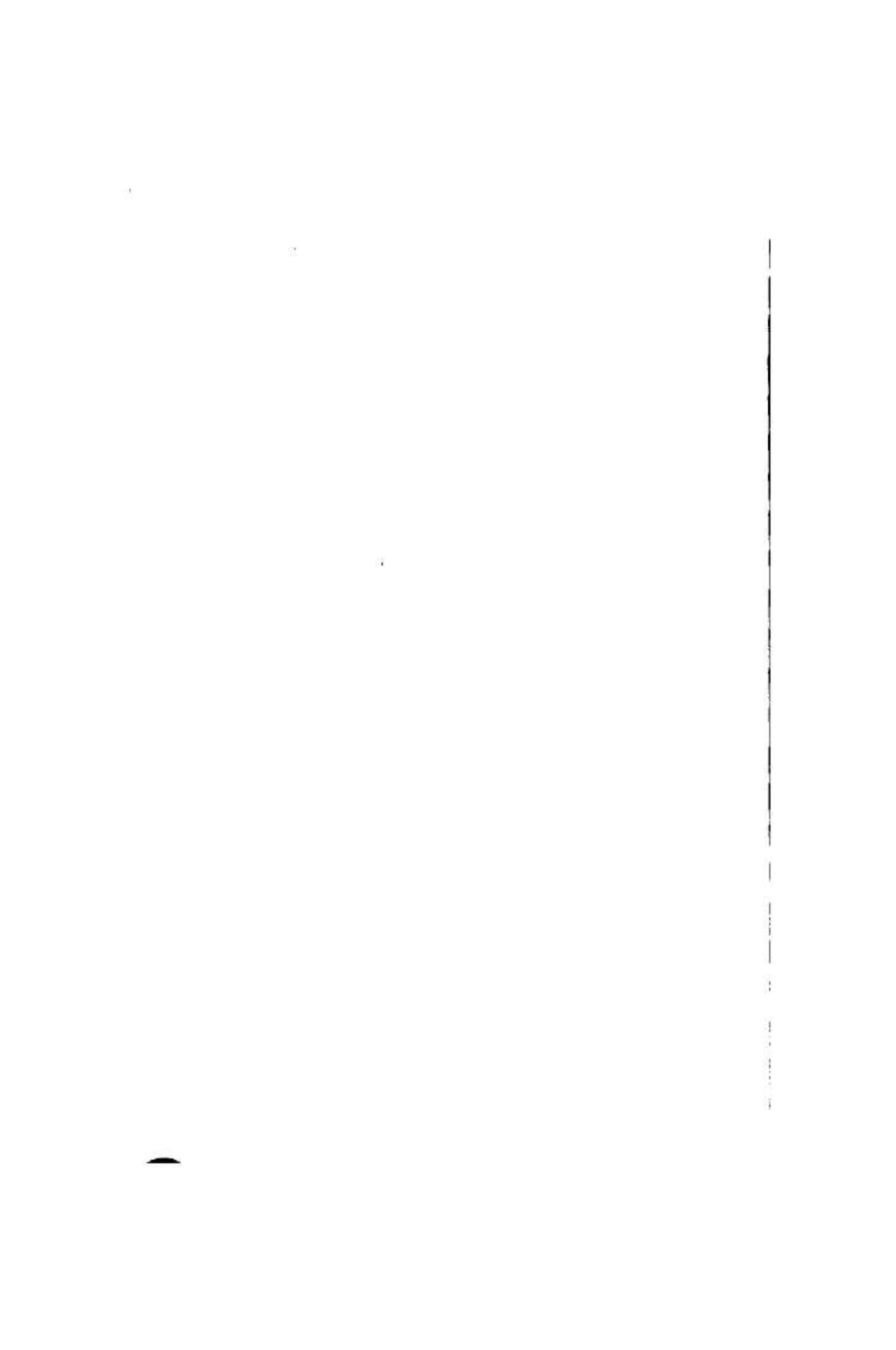
TO

**FABIAN FRANKLIN, Ph.D.,**

FORMERLY PROFESSOR OF MATHEMATICS IN THE JOHN HOPKINS UNIVERSITY,

**In Grateful Acknowledgment**

OF BENEFITS CONFERRED UPON THE AUTHOR DURING HIS RESIDENCE IN  
BALTIMORE AS A GRADUATE STUDENT OF THE  
JOHN HOPKINS UNIVERSITY,



## PREFACE.

THE present work is the result of long meditations and of an earnest search for truth. A conviction that truth in mathematics must be absolute, not admitting of any compromises, and an inmost feeling that Nature is not deceiving us and that She reveals Herself to us in Her true appearance, unmutilated by false logic, have guided me in my endeavors to solve one of the hardest mathematical problems, so intimately connected with the problem of the origin of our ideas,—namely, the problem of the Foundations of Geometry. These meditations were finally written up, in April, 1899, at the prompting of my excellent and highly esteemed friend and benefactor, Dr. Fabian Franklin, formerly Professor of Mathematics in the Johns Hopkins University, to whom I have availed myself of the present opportunity of expressing my gratitude, by inscribing this work to him.

Another name I ought to mention with gratitude is that of Dr. Alexander S. Chessin, Professor of Mathematics in Washington University, who was the first to appreciate the value of this work and to talk to me unreservedly about it, and also to urge me to use it as a thesis for the Ph.D. degree.

And, finally, I owe a duty of gratefulness to my distinguished professor, Dr. Frank Morley, for his guidance in the work of reading up the literature of the subject, for discussing with me points of difficulty in the literature, and also for allowing me to present some of my theorems before the conference of the mathematical seminary of the university, where the general discussion by the audience helped me in improving the mode of presentation of these theorems. To this discussion I owe, in particular, the analytical presentation of my proof that, *with the point as its element*, space must be three-dimensional. The whole of the Introduction was undertaken and carried out, during the academic year of 1900–1901, at the suggestion and under the direction of Professor Morley, and it is intended as a critical review of some of the most important results obtained by modern



mathematicians in the subject (Riemann, Beltrami, Lie and Poincaré), so that in the light of these an adequate estimate of the results achieved in this Dissertation might become possible.

And last, but not least, my thanks are due to Professor Edwin R. A. Seligman and Professor Felix Adler of Columbia University for the generous interest they have taken in the publication of my work in full; and also to Mr. Isador Goetz, A.B., of New York City, for his valuable assistance in the revision of the proof-sheets, and to the gentlemen of The New Era Printing Company for the care and efficiency with which the printing of this volume has been executed.

130 HENRY STREET, NEW YORK CITY,  
November, 1908.

## CONTENTS.

	PAGE.
INTRODUCTION .....	1-37
An infinity of mutually contradictory geometries postulated by Klein, Killing, and others — and its criticism.....	1-5
Sophus Lie's opinion as to the possibility of an efficient system of postulates and axioms of geometry.....	5-6
Lie's treatment of the Riemann-Helmholtz space-problem by the theory of transformation-groups, and the axioms which he assumes for this purpose. A comparison of this set of axioms with those assumed in the Dissertation.....	6-9
The interpretation of the non-Euclidian groups of displacements represents but a partially solved problem.....	10-11
Poincaré's opinions of the relation of pure reason and experience to the formation of our geometrical notions; his opinions as to the number of dimensions of space.....	11-16
An account of Riemann's Inaugural Dissertation on the Foundations of geometry and multiply-extended manifolds. — Résumé.....	16-20
Importance of the problem from a purely mathematical point of view. Theory of proportion and similar figures impossible in the non-Euclidian geometry; the squaring of the circle possible.....	20-21
A summary and justification of the author's views in his treatment of the subject. The conception of space as a number manifold of three dimensions — to be sought in the properties of rigidity, impenetrability, and infinite divisibility of material substance. The notion of distance derived from rigidity. The distance-line, or straight line, must be constructed in space, not in a plane, and deduced from the notion of distance. The plane should be constructed from the straight line, and both should be demonstrated to have all properties commonly attributed to them without justification. For instance, — the legitimacy of the assumption, of the plane being capable to move upon itself in a triply-infinite number of ways, and of its coincidence with itself when its two sides are interchanged — must follow from its construction.....	21-25
An account of Beltrami's views of the interpretability of the geometry of Lobatchevski upon the pseudosphere, based upon the analogy of the straight line and a geodesic upon that surface, when bending without stretching is allowed.....	26-27
Distinction emphasized between the straight line, as a figure capable of lying in a plane, and its notion as a geodesic in a	

	PAGE.
plane, — and reason given why in the later case the Euclidian postulate could not be proved . . . . .	27-29
Quotations from Bianchi, "Lezioni di geometria differenziale," concerning the futility of all attempts of proving the Euclidian postulate, and a justification of the author's attempt to do it by means of the <i>immaterial quadrilateral</i> , not bound to lie in a plane . . . . .	29-32
The identity of the author's postulates with those necessary for defining the group of displacements in general. A comparison with the postulates assumed by Helmholtz . . . . .	32
The interpretation of the non-Euclidian geometries again, and Beltrami's opinion that the non-Euclidian geometry of three dimensions is interpretable only analytically. — An allusion to a probable interpretation in line-space, with certain special conventions as to the meaning of metrical terms . . . . .	32-37

## DISSERTATION.

## THE FOUNDATIONS OF THE EUCLIDIAN GEOMETRY. 33-115

## CHAPTER I.

## (INTRODUCTORY.)

## SPACE AND ITS DIMENSIONS.

Definitions 1-6:—Geometry; space, extension; impenetrability, geometrical place; vacant space, infinite divisibility; magnitude and form; geometrical solid, volume . . . . .	38-39
Postulate 1,—Motion without distortion. Definitions 7-8:—Geometrical equality, coincidence; non-equality. Scholium and Résumé:—A rational justification of the definitions and postulate . . . . .	40-43
Definitions 9-12 and Résumé:—Distance and contact between bodies; Surface physical and geometrical; Portions and equality, or coincidence, of surfaces; Homogeneous and non-homogeneous surfaces . . . . .	44-47
Definitions 13-15:—Line; Portions, equality of lines, homogeneous lines; The point . . . . .	48-50
Résumé:—Discussion of the tri-dimensionality of space . . . . .	50-53
Thirteen paragraphs purporting to give an <i>analytical</i> discussion of the dimensions of space . . . . .	54-62

## CHAPTER II.

## THE SPHERE, CIRCLE, STRAIGHT LINE, ANGLE, TRIANGLE, PLANE, ETC.

Definition I,—Distance. Axiom I,—Continuity in congruence. Lemma 1,—A fixed body . . . . .	63-64
--	-------