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ROBERT D. CARMICHAEL

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DIOPHANTINE ANALYSIS

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PREFACE

THE author's purpose in writing this book has been to supply the reader with a convenient introduction to Diophantine Analysis. The choice of material has been determined by the end in view. No attempt has been made to include all special results, but a large number of them are to be found both in the text and in the exercises. The general theory of quadratic forms has been omitted entirely, since that subject would require a volume in itself. The reader will therefore miss such an elegant theorem as the following: Every positive integer may be represented as the sum of four squares. Some methods of frequent use in the theory of quadratic forms, in particular that of continued fractions, have been left out of consideration even though they have some value for other Diophantine questions. This is done for the sake of unity and brevity. Probably these omissions will not be regretted, since there are accessible sources through which one can make acquaintance with the parts of the theory excluded.

For the range of matter actually covered by this text there seems to be no consecutive exposition in existence at present in any language. The task of the author has been to systematize, as far as possible, a large number of isolated investigations and to organize the fragmentary results into a connected body of doctrine. The principal single organizing idea here used and not previously developed systematically in the literature is that connected with the notion of a multiplicative domain introduced in Chapter II.

The table of contents affords an indication of the extent and arrangement of the material embodied in the work.

Concerning the exercises some special remarks should be made. They are intended to serve three purposes: to afford practice material for developing facility in the handling of problems in Diophantine analysis; to give an indication of what special results have already been obtained and what special problems have been found amenable to attack; and to point out unsolved problems which are interesting either from their elegance or from their relation to other problems which already have been treated.

Corresponding roughly to these three purposes the problems have been divided into three classes. Those which have no distinguishing mark are intended to serve mainly the purpose first mentioned. Of these there are 133, of which 45 are in the Miscellaneous Exercises at the end of the book. Many of them are inserted at the end of individual sections with the purpose of suggesting that a problem in such position is readily amenable to the methods employed in the section to which it is attached. The harder problems taken from the literature of the subject are marked with an asterisk; they are 53 in number. Some of them will serve a disciplinary purpose; but they are intended primarily as a summary of known results which are not otherwise included in the text or exercises. In this way an attempt has been made to gather up into the text and the exercises all results of essential or considerable interest which fall within the province of an elementary book on Diophantine analysis; but where the special results are so numerous and so widely scattered it can hardly be supposed that none of importance has escaped attention. Finally those exercises which are marked with a dagger (35 in number) are intended to suggest investigations which have not yet been carried out so far as the author is aware. Some of these are scarcely more than exercises, while others call for investigations of considerable extent or interest.

ROBERT D. CARMICHAEL.

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