ELEMENTS OF GEOMETRY, CONTAINING THE FIRST SIX BOOKS OF EUCLID, WITH A SUPPLEMENT ON THE QUADRATURE OF THE CIRCLE, AND THE GEOMETRY OF SOLIDS; TO WHICH ARE ADDED, ELEMENTS OF PLANE AND SPHERICAL TRIGNONOMETRY

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Elements of geometry, containing the first six books of Euclid, with a supplement on the quadrature of the circle, and the geometry of solids; to which are added, Elements of plane and spherical trignonometry by Philip Kelland

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PHILIP KELLAND

ELEMENTS OF GEOMETRY, CONTAINING THE FIRST SIX BOOKS OF EUCLID, WITH A SUPPLEMENT ON THE QUADRATURE OF THE CIRCLE, AND THE GEOMETRY OF SOLIDS; TO WHICH ARE ADDED, ELEMENTS OF PLANE AND SPHERICAL TRIGNONOMETRY

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PREFACE TO THE ELEVENTH EDITION.

THE first six books of the present Treatise are precisely the first six books of Euclid's Elements. No alterations whatever have been made in the arrangement of the propositions, nor any of importance in the demonstration of those of the first four and sixth books. The same does not apply to the fifth book. The doctrine of Proportion laid down by Euclid in that book is an admirable specimen of reasoning based on an abstract definition. In simplicity of treatment, and in rigour of demonstration, this book leaves nothing to be desired. But the geometrical representation of what is essentially an arithmetical multiplication, renders the doctrine, as Euclid delivered it, somewhat difficult to be mastered. In the present treatise this difficulty has been obviated by the introduction of the concise language of algebra, whereby the reasoning is condensed and simplified, whilst the character of the demonstration remains unchanged. By this means the steps of the argument are brought near to one another, and the force of the whole is so clearly and distinctly perceived, that no more difficulty should be experienced in understanding the propositions of the fifth book than those of any other book of the Elements.

The Supplement consists of three books. The First Book treats of the rectification and quadrature of the circle. In the present edition, this book has been condensed and simplified.

The Second Book treats of the intersections of planes, and contains the most important propositions of the Eleventh Book of Euclid.

The Third Book treats of Solids, and exhibits, in a simple form, the most important propositions of the Twelfth Book of Euclid.

The treatise on Plane Trigonometry has, in the present edition, been increased by an additional section, containing some numerical examples, with a popular account of the nature and application of logarithms.

The treatise on Spherical Trigonometry, and the Notes, are reprinted with little alteration from the last edition.

The whole work is now so well known and appreciated, that a detailed explanatory preface is altogether superfluous.

PHILIP KELLAND.

COLLEGE OF EDINBURGH,
June 1, 1859.

ELEMENTS OF GEOMETRY.

BOOK FIRST.

DEFINITIONS.

I. A *point* is that which has position, but not magnitude.*

II. A *line* is length without breadth.

COROLLARY. The extremities of a line are points; and the intersections of one line with another are also points.

III. If two lines are such that they cannot coincide in any two points, without coinciding altogether, each of them is called a *straight* line.

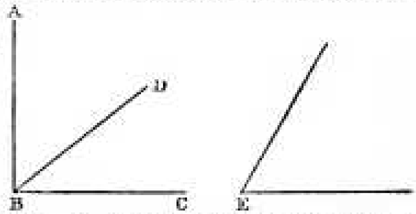
COR. Hence, two straight lines cannot enclose a space. Neither can two straight lines have a common segment; for they cannot coincide in part, without coinciding altogether.

IV. A *superficies* is that which has only length and breadth.

COR. The extremities of a superficies are lines; and the intersections of one superficies with another are also lines.

V. A *plane superficies* is that in which any two points being taken, the straight line between them lies wholly in that superficies.

VI. A *plane rectilineal angle* is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

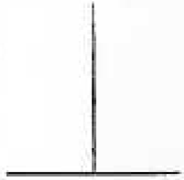


N.B.—When several angles are at one point B, any one of them is expressed by three letters, of which the letter that is at the vertex of the angle, that is, at the point in which the straight lines that contain the angle meet one another, is put between the other two letters, and one of these two is somewhere

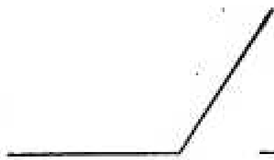
* See Notes.

upon one of those straight lines, and the other upon the other line: Thus the angle which is contained by the straight lines AB, CB, is named the angle ABC, or CBA; that which is contained by AB, BD, is named the angle ABD, or DBA; and that which is contained by DB, CB, is called the angle DBC, or CBD; but if there be only one angle at a point, it may be expressed by a letter placed at that point; as the angle at E.

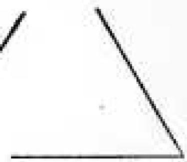
VII. When a straight line standing on another straight line makes the adjacent angles equal to one another, each of the angles is called a *right angle;* and the straight line which stands on the other is called a *perpendicular* to it.



VIII. An *obtuse angle* is that which is greater than a right angle.

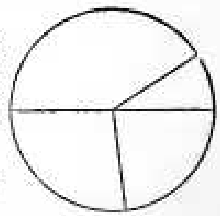


IX. An *acute angle* is that which is less than a right angle.



X. A *figure* is that which is enclosed by one or more boundaries. The space contained within a figure is called the *Area* of the Figure.

XI. A *circle* is a plane figure contained by one line, which is called the *circumference*, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal to one another.



XII. This point is called the *centre* of the circle.

XIII. A *diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

XIV. A *semicircle* is the figure contained by a diameter and the part of the circumference cut off by the diameter.

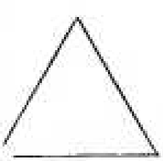
XV. *Rectilineal* figures are those which are contained by straight lines.

XVI. *Trilateral* figures, or *triangles*, by *three* straight lines.

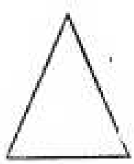
XVII. *Quadrilateral*, by *four* straight lines.

XVIII. *Multilateral* figures, or *polygons*, by more than four straight lines.

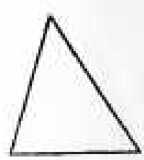
XIX. Of three-sided figures, an *equilateral* triangle is that which has three equal sides.



XX. An *isosceles* triangle is that which has (only) two sides equal.

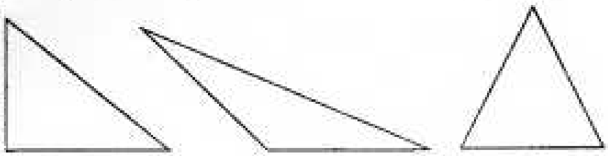


XXI. A *scalene* triangle is that which has three unequal sides.



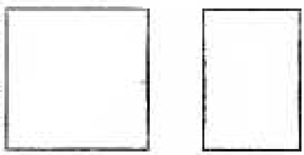
XXII. A *right-angled* triangle is that which has a right angle.

XXIII. An *obtuse-angled* triangle is that which has an obtuse angle.



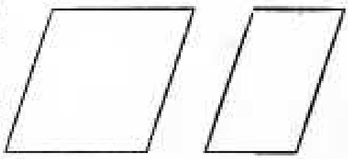
XXIV. An *acute-angled* triangle is that which has three acute angles.

XXV. Of four-sided figures, a *square* is that which has all its sides equal, and all its angles right angles.



XXVI. An *oblong* is that which has all its angles right angles, but has not all its sides equal.

XXVII. A *rhombus* is that which has all its sides equal, but its angles are not right angles.



XXVIII. A *rhomboid* is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.

XXIX. All other four-sided figures besides these are called *Trapeziums.*

XXX. Straight lines which are in the same plane, and being produced ever so far both ways, do not meet, are called *Parallel* Lines.

POSTULATES.

I. Let it be granted that a straight line may be drawn from any one point to any other point.

II. That a terminated straight line may be produced to any length in a straight line.

III. And that a circle may be described from any centre, at any distance from that centre.

AXIOMS.

I. Things which are equal to the same thing are equal to one another.*

II. If equals be added to equals, the wholes are equal.

III. If equals be taken from equals, the remainders are equal.

IV. If equals be added to unequals, the wholes are unequal.

V. If equals be taken from unequals, the remainders are unequal.

VI. Things which are doubles of the same thing, are equal to one another.

* See Notes.

VII. Things which are halves of the same thing, are equal to one another.

VIII. Magnitudes which coincide with one another; that is, which exactly fill the same space, are equal to one another.

IX. The whole is greater than its part.

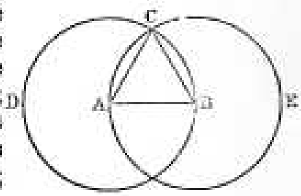
X. All right angles are equal to one another.

XI. Two straight lines which intersect one another, cannot be both parallel to the same straight line.

PROPOSITION I. PROBLEM.

To describe an equilateral triangle upon a given finite straight line.

Let AB be the given straight line; it is required to describe an equilateral triangle upon it.



From the centre A, at the distance AB, describe (Postulate 3) the circle BCD; and from the centre B, at the distance BA, describe the circle ACE; and from the point C, in which the circles cut one another, draw the straight lines (Post. 1) CA, CB, to the points A, B; ABC is an equilateral triangle.

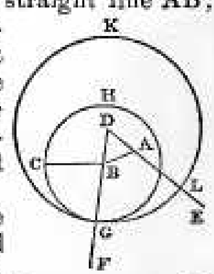
Because the point A is the centre of the circle BCD, AC is equal (Definition 11) to AB; and because the point B is the centre of the circle ACE, BC is equal to AB: But it has been proved that CA is equal to AB; therefore CA, CB, are each of them equal to AB; now, *things which are equal to the same thing are equal to one another* (Axiom 1); therefore CA is equal to CB; wherefore CA, AB, CB, are equal to one another; and the triangle ABC is therefore equilateral, and it is described upon the given straight line AB. Which was required to be done.

PROP. II. PROB.

From a given point to draw a straight line equal to a given straight line.

Let A be the given point, and BC the given straight line; it is required to draw, from the point A, a straight line equal to BC.

From the point A to B draw (Post. 1) the straight line AB; and upon it describe (I. 1) the equilateral triangle DAB, and produce (Post. 2) the straight lines DA, DB, to E and F; from the centre B, at the distance BC, describe (Post. 3) the circle CGH, and from the centre D, at the distance DG, describe the circle GKL. The straight line AL is equal to BC.



Because the point B is the centre of the circle CGH, BC is equal (Def. 11) to BG; and because D is the centre of the circle GKL, DL is equal to DG, and DA, DB, parts of them, are equal