DEAN'S STEREOTYPE EDITION. AN INTRODUCTION TO ALGEBRA: WITH NOTES AND OBSERVATIONS: DESIGNED FOR THE USE OF SCHOOLS AND PLACES OF PUBLIC EDUCATION

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JOHN BONNYCASTLE & JAMES RYAN

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ON THE

APPLICATION OF ALGEBRA TO GEOMETRY.

BY JOHN BONNYCASTLE,

Professor of Mathematics in the Royal Military Academy, Woolwich.

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PREFACE.

THE powers of the mind, like those of the body, are increased by frequent exertion; application and industry supply the place of genius and invention; and even the creative faculty itself may be strengthened and improved by use and perseverance. Uncultivated nature is uniformly rude and imbecile, it being by imitation alone that we at first acquire knowledge, and the means of extending its bounds. A just and perfect acquaintance with the simple elements of science, is a necessary step towards our future progress and advancement; and this assisted by laborious investigation and habitual inquiry, will constantly lead to eminence and perfection.

Books of rudiments, therefore, concisely written, well digested, and methodically arranged, are treasures of inestimable value; and too many attempts cannot be made to render them perfect and complete. When the first principles of any art or science are firmly fixed and rooted in the mind, their application soon becomes easy, pleasant and obvious; the understanding is delighted and enlarged; we conceive clearly, reason distinctly, and form just and satisfactory conclusions. But, on the contrary, when the mind, instead of reposing on the stability of truth and received principles, is wandering in doubt and uncertainty, our ideas will necessarily be confused and obscure; and every step we take must be attended with fresh difficulties and endless perplexity.

That the grounds, or fundamental parts of every science, are dull and unentertaining, is a complaint universally made, and a truth not to be denied; but then, what is obtained with difficulty is usually remembered with ease; and what is purchased with pain is often possessed with pleasure. The seeds of knowledge are sown in every soil, but it is by proper culture alone that they are cherished and brought to maturity. A few years of early and assiduous application never fails to procure us the reward of our industry; and who is there, who knows the pleasures and advantages which the sciences afford, that would think his time, in this case, misspent, or his labours useless? Riches and honours are the gift of fortune, casually bestowed, or hereditarily received, and are frequently abused by their possessors; but the superiority of wisdom and knowledge is a pre-eminence of merit, which originates with the man, and is the noblest of all distinctions.

Nature, bountiful and wise in all things, has provided us with an infinite variety of scenes, both for our instruction and entertainment; and, like a kind and indulgent parent, admits all her children to an equal participation of her blessings. But as the modes, situations, and circumstances of life are various, so accident, habit, and education, have each their predominating influence, and give to every mind its partic-

ular bias. Where examples of excellence are wanting, the attempts to attain it are but few; but eminence excites attention, and produces imitation. To raise the curiosity, and to awaken the listless and dormant powers of younger minds, we have only to point out to them a valuable acquisition, and the means of obtaining it; the active principles are immediately put into motion, and the certainty of the conquest is ensured from a determination to conquer.

But of all the sciences which serve to call forth this spirit of enterprise and inquiry, there are none more eminently useful than Mathematics. By an early attachment to these elegant and sublime studies, we acquire a habit of reasoning, and an elevation of thought, which fixes the mind, and prepares it for every other pursuit. From a few axioms, and evident principles, we proceed gradually to the most general propositions and remote analogies; deducing one truth from another, in a chain of argument well connected and logically pursued; which brings us at last, in the most satisfactory manner, to the conclusion, and serves as a general direction in all our inquiries after truth.

And it is not only in this respect that mathematical learning is so highly valuable; it is, likewise, equally estimable for its practical utility. Almost all the works of art and devices of man, have a dependance upon its principles, and are indebted to it for their origin and perfection. The cultivation of these admirable sciences is, therefore, a thing of the utmost importance, and ought to be considered as a principal part of every liberal and well-regulated plan of education. They are the guide of our youth, the perfection of our reason, and the foundation of every great and noble undertaking.

From these considerations, I have been induced to compose an introductory course of mathematical science; and from the kind encouragement which I have hitherto received, am not without hopes of a continuance of the same candour and approbation. Considerable practice as a teacher, and a long attention to the difficulties and obstructions which retard the progress of learners in general, have enabled me to accommodate myself the more easily to their capacities and understandings. And as an earnest desire of promoting and diffusing useful knowledge is the chief motive for this undertaking, so no pains or attention shall be wanting to make it as complete and perfect as possible.

The subject of the present performance is ALGEBRA; which is one of the most important and useful branches of those sciences, and may be justly considered as the key to all the rest. Geometry delights us by the simplicity of its principles, and the elegance of its demonstrations; Arithmetic is confined in its object, and partial in its application; but Algebra, or the Analytic Art, is general and comprehensive, and may be applied with success in all cases where truth is to be obtained and proper data can be established.

To trace this science to its birth, and to point out the various alterations and improvements it has undergone in its progress, would far exceed the limits of a preface.* It will be sufficient to observe that the invention is of the highest antiquity, and has challenged the praise and adoration of all ages. *Diophantus*, a Greek mathematician, of Alexandria in Egypt, who flourished in or about the third century after Christ, appears to have been the first, among the ancients, who applied it to the solution of indeterminate or unlimited problems; but it is to the moderns that we are principally indebted for the most curious refinements of the art, and its great and extensive usefulness in every abstruse and difficult inquiry. *Newton, Maclaurin, Sanderson, Simpson,*

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* Those who are desirous of a knowledge of this kind, may consult the Introduction to my *Treatise on Algebra*; where they will find a regular historical detail of the rise and progress of the science, from its first rude beginnings to the present times.