# ELEMENTS OF DESCRIPTIVE GEOMETRY, PART I, ORTHOGRAPHIC PROJECTIONS

Published @ 2017 Trieste Publishing Pty Ltd

#### ISBN 9780649007950

Elements of descriptive geometry, part I, Orthographic projections by Albert E. Church & George M. Bartlett

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd. Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

### **ALBERT E. CHURCH & GEORGE M. BARTLETT**

# ELEMENTS OF DESCRIPTIVE GEOMETRY, PART I, ORTHOGRAPHIC PROJECTIONS



## ELEMENTS

OF

## DESCRIPTIVE GEOMETRY

# PART I ORTHOGRAPHIC PROJECTIONS

BY

ALBERT E. CHURCH, LLD.

LATE PROFESSOR OF KATHEMATICS IN THE UNITED STATES -MILITARY AUADEMY

AND

GEORGE M. BARTLETT, M.A.

INSTRUCTOR IN DESCRIPTIVE GROWERS AND MECHANISM

NEW YORK -: CINCINNATI -: CHICAGO AMERICAN BOOK COMPANY Corrasont, 1964, pr BARNES & BURR,

Сомпания, 1884, ку А. э. ВАВNES & CO.

COPTROUDT, 1593 AND 1992, RV MARGARET A BLUNT.

CONVEGET, 1913, BY GEORGE M. BARTLETT.

ENTERED AT STATIOTIONS HALL, LONDON,

O. O. DESCR. GRAM. PAGE C. W. P. I



#### PREFACE

Church's "Elements of Descriptive Geometry" was originally published in 1864. The preface to the first edition states: "Without any effort to enlarge or originate, the author has striven to give, with a natural arrangement and in clear and concise language, the elementary principles and propositions of this branch of science, of so much interest to the mathematical student, and so necessary to both the civil and military engineer."

Professor Church succeeded so well in his efforts to produce a practical and well-adapted treatise that it has continued in use as a text-book for more than forty years in the United States Military Academy and in many other academics, technical schools, and colleges. This long-continued use of the book speaks well for its high intrinsic excellence.

During the last few years, however, there have taken place many changes in the methods of teaching the subject, and in the problems required. To meet these new demands the present volume is issued. In its preparation much of Professor Church's text has been used, and his concise and lucid style has been preserved.

Among the salient features of the present work are the following:

The figures and text are included in the same volume.

General cases are preferred to special ones.

A sufficient number of problems are solved in the third angle to familiarize the student with its use. A treatment of the profile plane of projection is introduced.

Many exercises for practice have been introduced.

Several new problems have been added.

The old figures have been redrawn, and many of them have been improved.

Several of the more difficult elementary problems have been illustrated by pictorial views.

In the treatment of curved surfaces, all problems relating to single-curved surfaces are taken up first, then those relating to warped surfaces, and finally those relating to surfaces of revolution. Experience proves this order to be a logical one, as we here proceed "from the simple to the more complex." Also the student is more quickly prepared for drawing-room work on intersections and developments; and in case it is desired to abbreviate the course by omitting warped surfaces, the remaining problems will be found to be consecutively arranged.

The writer here wishes to acknowledge his indebtedness to the many teachers who have aided him with valuable advice and suggestions in relation to this work. In particular his thanks are due to his esteemed colleagues, Professor H. J. Goulding and Mr. D. E. Foster of the University of Michigan, for their careful reading and correction of the manuscript.

G. M. B.

MAY 14, 1910.

## CONTENTS

### PART I

Preliminary Definitions	ORTHOG	RA	PHIC	1.13	OJE	CTR	NS				
Representation of Pianes Representation of Pianes Representation of Straight Lines Representation of Straight Lines Propositions relating to the Point, Line, and Plane Rotation of the Horizontal Plane Notation used in the Description of Brawings Exercises for Practice The Profile Plane of Projection Elementary Problems relating to the Point, Line, and Plane Classification of Lines Projection of Curves Tangents and Normals to Lines Construction of Certain Plane Curves The Helix, Generation and Properties Construction of Certain Plane Curves The Helix, Generation and Properties Conical Surfaces, Generation and Properties Planes Tangent to Surfaces in Generat Conical Surfaces, Generation and Properties Repoints in which Surfaces are pierced by Lines Intersection of Cylinders and Cones, Developments Convolutes, and Problems relating to Them Warped Surfaces with a Plane Directer The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Directer The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Directer The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Directer The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Directer The Hyperboloid of Revolution The Hyperboloid of Revolution The Hyperboloid of Revolution The Hyperboloid of Revolution Planes Tangent to Surfaces of Revolution	D . U										PAGE
Representation of Planes Representation of Straight Lines Propositions relating to the Point, Line, and Plane Rotation of the Horizontal Plane Rotation used in the Description of Drawings Exercises for Practice The Profile Plane of Projection Elementary Problems relating to the Point, Line, and Plane Classification of Lines Projection of Curves Tangents and Normals to Lines Construction of Certain Plane Curves The Helix. Generation and Properties. The Helix. Generation and Properties Cylindrical Surfaces. Generation and Properties Conical Surfaces. Generation and Properties Planes Tangent to Surfaces in General Planes Tangent to Cylinders and Comes Points in which Surfaces are pierced by Lines Intersection of Cylinders and Comes. Developments Convolutes, and Problems relating to Them Warped Surfaces with a Plane Director The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Director The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Director The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Director The Hyperbolic Paraboloid Planes Tangent to Surfaces of Revolution The Hyperboloid of Hevolution of One Nappe Double-Curved Surfaces of Revolution The Hyperboloid of Hevolution of One Nappe Double-Curved Surfaces of Revolution Planes Tangent to Surfaces of Revolution		.+	96	100		*:			*		- 0
Representation of Straight Lines		Š	**	1	*	ŧ.			*	4	- 1.5
Propositions relating to the Point, Line, and Plane Rotation of the Horizontal Plane Rotation used in the Description of Drawings Exercises for Practice The Profile Plane of Projection Elementary Problems relating to the Point, Line, and Plane Classification of Lines Projection of Curves Taugents and Normals to Lines Construction of Certain Plane Curves The Helix Generation and Properties Generation and Classification of Surfaces Cylindrical Surfaces. Generation and Properties Conical Surfaces. Generation and Properties Planes Tangent to Surfaces in General Planes Tangent to Cylinders and Comes Points in which Surfaces are pierced by Lines Intersection of Cylinders and Comes. Developments Convolutes, and Problems relating to Them Warped Surfaces with a Plane Directer The Hyperbolic Paraholoid Planes Tangent to Warped Surfaces with a Plane Directer The Helicoid Warped Surfaces with Three Linear Directrices Surfaces of Revolution The Hyperbolic of Herolution of One Nappe Double Curved Surfaces of Revolution The Hyperbolic of Surfaces of Revolution Planes Tangent to Surfaces of Revolution Planes Tangent to Surfaces of Revolution The Hyperbolic of Surfaces of Revolution		is Naces	æ.	6		*		*			
Rotation of the Horizontal Plane 17 Notation used in the Description of Drawings 19 Exercises for Practice 21 The Profile Plane of Projection 224 Elementary Problems relating to the Point, Line, and Plane 28 Classification of Lines 25 Projection of Curves 36 Tangents and Normals to Lines 36 Construction of Certain Plane Curves 37 The Helix Generation and Properties 37 Generation and Classification of Surfaces 37 Conical Surfaces Generation and Properties 37 Conical Surfaces Generation and Properties 38 Planes Tangent to Surfaces in General 38 Planes Tangent to Cylinders and Cones 38 Points in which Surfaces are pierced by Lines 38 Intersection of Cylinders and Cones 38 Convolutes, and Problems relating to Them 32 Warped Surfaces with a Plane Directer 32 The Hyperbolic Paraholoid 32 Planes Tangent to Warped Surfaces with a Plane Directer 32 The Helicoid 32 Warped Surfaces with Three Linear Directrices 34 Surfaces of Revolution 34 Double Curved Surfaces of Revolution 35 Intersection of Surfaces of Revolution 35 Intersection of Surfaces of Revolution 35 Intersection of Surfaces of Revolution 36 Intersection of Surfaces 37 Intersection							11.54		*		
Notation used in the Description of Drawings						ue	•	*	*	-	
Exercises for Practice											
The Profile Plane of Projection  Elementary Problems relating to the Point, Line, and Plane  Classification of Lines  Projection of Curves  Tangents and Normals to Lines  Construction of Certain Plane Curves  The Helix, Generation and Properties  Generation and Classification of Surfaces  Cylindrical Surfaces, Generation and Properties  Conical Surfaces, Generation and Properties  Conical Surfaces, Generation and Properties  Planes Tangent to Surfaces in Generat  Planes Tangent to Cylinders and Cones  Points in which Surfaces are pierced by Lines  Intersection of Cylinders and Cones.  Developments  Convolutes, and Problems relating to Them  Warped Surfaces with a Plane Directer  The Hyperbolic Paraboloid  Planes Tangent to Warped Surfaces with a Plane Directer  The Helicoid  Warped Surfaces with Three Linear Directrices  Surfaces of Revolution  The Hyperboloid of Revolution of One Nappe  147  Double-Curved Surfaces of Revolution  Planes Tangent to Surfaces of Revolution  Intersection of Surfaces of Revolution				rate to	ngs	*		*	*	39	
Elementary Problems relating to the Point, Line, and Plane Classification of Lines				4		+0	14	+	+		111
Classification of Lines						*:		*	***	2	
Projection of Curves Tangents and Normals to Lines Construction of Certain Plane Curves The Helix, Generation and Properties The Helix, Generation and Properties Generation and Classification of Surfaces Cylindrical Surfaces, Generation and Properties The Conical Surfaces, Generation and Properties Conical Surfaces, Generation and Properties Thanes Tangent to Surfaces in Generat Planes Tangent to Cylinders and Cones Points in which Surfaces are pierced by Lines Intersection of Cylinders and Cones. Developments The Hyperbolic Paraboloid Planes Tangent to Warped Surfaces with a Plane Directer The Hyperbolic Paraboloid The Hyperbolic Paraboloid The Hyperboloid of Hevolution The Hyperboloid of Revolution The Hyperboloid of Revolution The Hyperboloid of Revolution The Hyperboloid of Revolution The Hyperboloid of Surfaces of Revolution		g 10	the F	dint	, Lin	e, au	d Pla	ne			
Tangents and Normals to Lines  Construction of Certain Plane Curves  The Helix, Generation and Properties  Generation and Classification of Surfaces  Cylindrical Surfaces, Generation and Properties  Conical Surfaces, Generation and Properties  Conical Surfaces, Generation and Properties  Stangent to Surfaces in Generat  Planes Tangent to Cylinders and Cones  Points in which Surfaces are pierced by Lines  Points in which Surfaces are pierced by Lines  Intersection of Cylinders and Cones, Developments  Convolutes, and Problems relating to Them  120  Warped Surfaces with a Plane Directer  The Hyperbolic Paraboloid  Planes Tangent to Warped Surfaces with a Plane Directer  The Helicoid  Warped Surfaces with Three Linear Directrices  Surfaces of Revolution  The Hyperboloid of Revolution of One Nappe  147  Double-Curved Surfaces of Revolution  Flanes Tangent to Surfaces of Revolution  Intersection of Surfaces of Revolution with Other Surfaces		+	240	133	95	183		3.5	80	3.5	
Construction of Certain Plane Curves	이번 물을 빼앗기 때에 본러가게 가셨다면 이번 생각이 되고 있어 있다면 나를 만든다면 하는데 되었다.	ž.,			87	0.0			23	4	
The Helix. Generation and Properties	[[[[[[[]]]] [[[]] [[]] [[]] [[] [[]] [[] [[]] [[] [[]] [[] []		2.1	120	121	2.0		:21	2.0	1.2	63
Generation and Classification of Surfaces	Construction of Certain Plane	C	trees		1	30			-		(35)
Cylindrical Surfaces. Generation and Properties. 78 Conical Surfaces. Generation and Properties 81 Planes Tangent to Surfaces in Generat 84 Planes Tangent to Cylinders and Cones 86 Points in which Surfaces are pierced by Lines 94 Intersection of Cylinders and Cones. Developments 86 Convolutes, and Problems relating to Them 120 Warped Surfaces with a Plane Director 125 The Hyperbolic Paraboloid 128 Planes Tangent to Warped Surfaces with a Plane Director 134 The Helicoid 138 Warped Surfaces with Three Linear Directrices 142 Surfaces of Revolution 140 The Hyperboloid of Hevolution of One Nappe 147 Double-Curved Surfaces of Revolution 156 Planes Tangent to Surfaces of Revolution 158 Intersection of Surfaces of Revolution 158 Intersection of Surfaces of Revolution 158 Intersection of Surfaces of Revolution 158	The Helix, Generation and	Pro	pertie	5.	(3)		4.				73
Conical Surfaces. Generation and Properties	Generation and Classification	of	Serfa	cea	3			•			76
Planes Tangent to Surfaces in General	Cylindrical Surfaces. General	al in	or med	Pro	pertie	× .		02	0.00	-	78
Planes Tangent to Cylinders and Cones	Conical Surfaces. Generation	11, 311	d Po	pert	ies	300	0.007		(40)	004	81
Points in which Surfaces are pierced by Lines	Planes Tangent to Surfaces in	G	eneral	11	14						84
Intersection of Cylinders and Cones. Developments	Planes Tangent to Cylinders	and	Come	N.	1000	34.0	10.0		4.0	2.0	86
Convolutes, and Problems relating to Them	Points in which Surfaces are	pier	ced b	v Li	ties		(a)				94
Warped Surfaces with a Plane Director	Intersection of Cylinders and	Co	nes.	Dev	elopm	ents	204	340	*00	1.4	240
The Hyperbolic Paraholoid	Convolutes, and Problems rel	atin	g to	Phen	п.					1	120
Planes Tangent to Warped Surfaces with a Plane Directer	Warped Surfaces with a Plan	e D	irecte								125
Planes Tangent to Warped Surfaces with a Plane Director	The Hyperbolic Paraholoid	4		63	-	1					128
The Helicoid		urfa	ices w	ith a	Plat	ie Di	recter	۲.			132
Surfaces of Revolution								400	- 23	-	138
Surfaces of Revolution	Warped Surfaces with Three	Lis	ear I	lirect	rices				-		142
The Hyperboloid of Revolution of One Nappe	# (Company of the Company of the Com					411		•	400	.04	146
Double-Curved Surfaces of Revolution									- 13		147
Planes Taugent to Surfaces of Revolution									***		156
Intersection of Surfaces of Revolution with Other Surfaces , 163					18	33	126	18	- 35		158
					Oth	er Su	rface	ă .			
									Soh	ri-	(500
cal Triangles			Brook		and the same				-	33	169



### PART 1

#### ORTHOGRAPHIC PROJECTIONS

#### PRELIMINARY DEFINITIONS

- 1 Geometry enables us to determine unknown magnitudes, relationships, and forms from those which are known. There are in general two methods of solution for any given problem; namely, the analytical and the graphical. In the former we arrive at our results by calculation; in the latter we make drawings which represent graphically the true relationships between the points, lines, and surfaces under consideration, and arrive at our results without calculation.
- 2. Graphics. If the problem relates to points and lines lying in only one plane, the graphical solution may be reached by a simple application of the principles of Geometrical Drawing, or Plane Graphics.

If the problem relates to magnitudes not in the same plane, the graphical solution would require an application of the principles of Descriptive Geometry, or the *Graphics of Space*.

3. Descriptive Geometry is that branch of Mathematics which has for its object the explanation of the methods of representing by drawings:

First. All geometrical magnitudes.

Second. The solution of problems relating to these magnitudes in space.

These drawings are so made as to present to the eye, situated at a particular point, the same appearance as the magnitude or object itself, were it placed in the proper position.