

**A SERIES OF FIGURES  
ILLUSTRATIVE OF GEOMETRICAL  
OPTICS, TOGETHER WITH AN  
EXPLANATION FORMING A  
TREATISE**

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A series of figures illustrative of geometrical optics, together with an explanation forming a treatise by W. B. Hopkins

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**W. B. HOPKINS**

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A SERIES OF FIGURES  
ILLUSTRATIVE OF  
GEOMETRICAL OPTICS

REDUCED FROM STEEL ENGRAVINGS EXECUTED BY P. ENGEL  
UNDER THE DIRECTION OF PROFESSOR K. SCHELLBACH OF BERLIN;

TOGETHER WITH AN EXPLANATION

FORMING

A TREATISE

TRANSLATED FROM THE GERMAN OF PROFESSOR SCHELLBACH.

The whole Edited,

WITH NOTES AND AN APPENDIX,

BY

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MATHEMATICAL LECTURER OF GONVILLE AND CAIUS COLLEGE, CAMBRIDGE.

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1851.

## P R E F A C E.

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THE Plates, which are the principal feature of this work, are lithographed copies of reduced drawings from a beautiful series of steel engravings which appeared last year in Berlin.

The description of the Plates is translated by the Editor's friend and pupil Mr. Tomkins of St. Catharine's Hall, from the German of Professor Schellbach. The notes and Appendix were added by the Editor.

Every Plate has been in a greater or less degree altered, chiefly for the purpose of making them less intricate and so better fitted for purposes of explanation, and more intelligible to a beginner. With this view some of the representations of eyes in different positions have been left out, as will be seen by referring to the notes. It appeared to the Editor that they might be dispensed with, because no new principle would be exemplified by retaining them. In the second and fifth Plates, and in figure 2 of the fourteenth, one-half of the original figures have been omitted; because the caustics are symmetrical with regard to the axis of each figure, and both sides are formed in precisely the same way. The modified figures shew the actual formation of the caustic on one side of the axis; and on the other side they indicate the way in which two rays intersect upon the caustic. In figures 1, 3, and 5 of the first Plate, two out of the three pencils proceeding from *A*, *B*, and *C*, are represented only in outline, because no

new principle is illustrated by giving them in full like the pencil from *A*. As a general rule one-half of the rays have been omitted from the figures which represent the modifications of pencils of light. The representations are thus relieved of much of their apparent complexity, whilst the graphic effect is undiminished.

The Appendix contains matter either important or likely to be generally interesting. At some future time its dimensions may be enlarged. At present the Editor does not think it necessary to add more; and he does not believe that there is room for a new treatise upon Optics.

It is hoped that the work will prove useful to English students. The original drawings, though unquestionably superior to these, were so expensive as to be beyond the reach of many students. The explanation also was in German: and this circumstance still more narrowly limited the number of persons who could derive benefit from the original work. A correspondence has been entered into with the Proprietors of that work, in order that they may be indemnified from any injury which the publication of this translation might entail; and it is believed that this object has been accomplished.

*S. Catharine's Hall, June 3, 1851.*

#### ERRATA.

Page 10, line 11, for *n* read *a*.

" 11, " 2, omit the word *about*.

# GEOMETRICAL OPTICS.

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## EXPLANATION OF THE PLATES.

THE object of these drawings is to facilitate the difficult study of Optics, and especially to give a clear idea of the working of Optical instruments.

That the common treatises, and even careful calculations, do not effect this object in the same degree as constructions like these, we have the testimony of men eminent in physical science, even if a bare inspection of these drawings did not at once confirm this statement.

For the explanation of them we suppose only an acquaintance with the well-known laws of reflexion and refraction.



## PLATE I.

*Figure 1.*

THE line  $LM$  represents the intersection of a reflecting plane with the plane of the paper at right angles. About this mirror is a perfectly uniform medium, within which, as is well known, rays of light are propagated in straight lines.

From the point  $A$  of this medium proceeds a pencil of rays, as  $AL$ ,  $AG$ ,  $AM$ , which are reflected from the mirror at the angle at which they are incident.

Thus, for instance, if the angle  $ALG = OLN$ , the ray  $AL$ , after reflexion, takes the course  $LO$ . The perpendicular ray  $AG$  is the only one which returns back upon itself, the rest after reflexion take a different course.

But if we suppose any one of these rays as  $LO$ , produced backwards, in the direction  $LA'$ , it is manifest, from the similarity of the triangles  $ALG$  and  $A'LG$ , that all the reflected rays so produced pass through a point  $A'$ , and thus return into the medium in the same way as if they proceeded from  $A'$ , a point at the same distance behind the mirror as the luminous point is before it. If we now conceive the entire figure to revolve about the line  $AA'$ , perpendicular to  $LM$ , then  $LM$  passes over the surface of the mirror, and the lines  $AL$ ,  $AM$  describe a cone whose vertex is  $A$ , while  $A'L$ ,  $A'M$  describe a cone whose vertex is  $A'$ .

If then the luminous point  $A$  fills the cone  $ALM$  with rays of light, all these rays will be reflected as if they proceeded from the vertex  $A'$ .

Suppose now an observer at  $E$  to look upon the mirror, his eye receives from it a small cone of rays whose vertex is  $A'$ , and whose base rests on the transparent cornea of the eye: here the rays undergo refraction, pass through the fluids of the eye, and after various other refractions converge upon the retina to the vertex of a cone, where they have sufficient power to stimulate

the optic nerve so as to give rise to the perception of sight. Thus the eye is affected just as if the rays had proceeded from the point  $A'$ , and therefore an observer at  $E$  not only sees the point  $A$  in the direction  $EHA'$ , but believes he sees it in the point  $A'$  itself.

The point  $A'$  may thus be called the *image* of  $A$ .

In speaking of a luminous point, it need scarcely be said that we do not mean a *mathematical* point; such a point would be no more visible than a mathematical line. By a luminous point we understand a minute portion of a luminous surface in no direction perceptibly extended. And so by a luminous line we understand such a surface perceptibly extended only in one direction.

In figure 1 two other points  $B, C$  are taken from which pencils of light proceed. The images of these points are observed in  $B', C'$ ;\* these images  $A', B', C'$  are all seen in the same position by any eye before the mirror. But this simplest case happens only when the mirror is plane.

Figure 2.

In this figure  $A, B, C$  is supposed to be a luminous object whose two extremities and middle point have the same position relative to the mirror as the points  $A, B, C$  in figure 1; the rays  $AH$  from  $A$ ,  $BI$  from  $B$ , and  $CK$  from  $C$ , come to the eye at  $E$ . Or rather the eye at  $E$  receives from the points  $A, B, C$  pencils of rays of which  $HE, IE, KE$  are the axes, and whose vertices would lie in  $A', B', C'$ , if produced beyond the mirror.

But from all points of the arrow  $ABC$  such cones of rays reach the eye, and therefore it sees an image of this arrow in  $A', B', C'$ . In like manner, eyes at  $D$  and  $F$  manifestly see the object  $ABC$  in exactly the same form, in the position  $A'B'C'$ , and at the same distance behind the mirror as the arrow is before it.

Figure 3.

In this figure we suppose the space below the line  $DE$  to be occupied by water, the space above by air.

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\* In the original plate all the rays from  $B$  and  $C$  are also drawn. The greater part are here omitted to avoid confusion.

From the luminous point  $A$  proceeds a pencil of rays  $DAE$ , which is refracted on emerging into the air.

Thus let  $AK$  be a ray of light, and  $AH$  a perpendicular to the surface  $DE$ , and suppose a normal drawn at  $K$ . Then  $KAH$  is the angle which the ray makes with the normal, i.e. it is the angle of *incidence*. If now the refracted ray  $KL$  be produced backwards to cut the perpendicular  $AH$  in  $O$ , then the angle  $KOH$  represents the angle of *refraction*.

The refractive index for water is nearly  $\frac{4}{3}$ ; so that we have  $\sin KAH : \sin KOH = 3 : 4$ ; or, since in the triangle  $KAO$  these sines are as  $KO : KA$ , we have  $KO : KA = 3 : 4$ . If then  $KA$  be divided into four equal parts, and with centre  $A$  and radius equal three of these parts a circle be described, it will cut the perpendicular in  $O$ . In order therefore to find the direction of the ray  $AK$  after refraction, we have only to produce the line  $OK$  to  $L$ .

In this way all the rays which emerge from the water may be accurately determined.\*

If all the rays proceeding from  $A$ , which fall between the legs of the angle  $DAE$ , could have been constructed, the emerging refracted rays produced backwards into the water, would have formed by their mutual intersection a continuous curve which is even here to be perceived in the figure  $DMA'E$ , and appears more fully developed in figure 4.

This curve to which the refracted rays are tangents, is one of the simplest kinds of *caustics*, which perform so important a part in Optics.

From the above construction of the refracted rays it appears also that the cusp  $A'$  of this caustic line lies  $\frac{1}{4}$ <sup>th</sup> part of the distance  $AH$  of the luminous point from the surface of the water higher than this point; for a ray incident close to the foot  $H$  of the perpendicular  $AH$ , will manifestly be so refracted as to intersect in  $A'$  if  $HA' = \frac{3}{4}HA$ . It is also manifest that the caustic line only extends to the points  $D$  and  $E$ , if  $DH = EH = \frac{3}{4}DA$ , for then the ray which emerges at  $D$  or

\* The author of the original work here remarks that "every single ray in these figures has been constructed with such care that the angular error is never greater than a few minutes."