NOTES ON ELECTRICITY AND MAGNETISM

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649738946

Notes on Electricity and Magnetism by R. C. Dearle & K. H. Kingdon

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

R. C. DEARLE & K. H. KINGDON

NOTES ON ELECTRICITY AND MAGNETISM

Trieste

NOTES

ON

ELECTRICITY AND MAGNETISM

BY

R. C. DEARLE, M.A.

AND

K. H. KINGDON, M.A.

UNIVERSITY OF TORONTO PRESS
TORONTO

ELECTRICITY.

1. Coulomb's Law.

If there be two point charges e_1 and e_2, separated by a distance r, the mutual force exerted between them will be $\frac{e_1 e_2}{r^2}$.

2. Coulomb's Torsion Balance.

This instrument was designed by Coulomb for verifying the truth of the above law. It consists of an insulating rod AB (Fig. 1a) suspended from a torsion head by a fine wire and having a small gilt pith-ball fixed at A. C is a similar pith-ball fixed at the end of an insulating rod. The balls are adjusted so as to touch each other and are then charged. The ball A is repelled through an angle θ_0 until the force of repulsion is balanced by the torsion in the suspending wire; the torsion is then a measure of the force of repulsion.

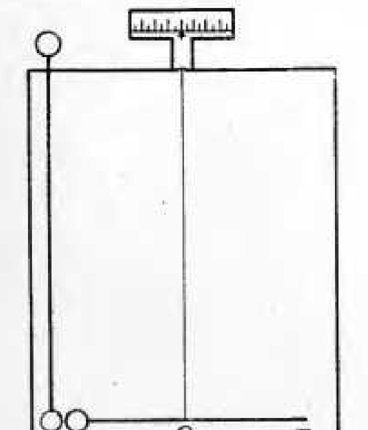

Fig 1a

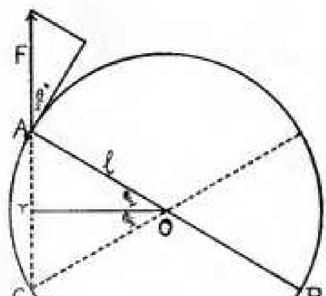

Fig 1b

According to Coulomb's law $Fr^2 = e_1 e_2 =$ a constant; hence we can test the truth of this law by increasing the torsion and observing whether the distance between the balls varies according to the law. Let the torsion head be turned back through $T°$ thereby reducing the angle subtended by the balls at 0 to θ, and making the torsion in the wire equal to $(T+\theta)$. Then from Fig. 1*b*, $CA = 2l \sin \frac{\theta}{2}$. The component of F, the force of repulsion between C and A, tending to twist the wire will be $F \cos \frac{\theta}{2}$, and the moment of this component about 0 is $Fl \cos \frac{\theta}{2}$. This moment is balanced by the torsion in the wire.

If K be the moment of the couple required to produce a torsion of $1°$ in the wire, for a torsion of $(T+\theta)°$ the couple will be $K(T+\theta)$.

$$\therefore\ Fl \cos \frac{\theta}{2} = K(T+\theta)$$

$$\therefore\ F = \frac{K(T+\theta)}{l \cos \frac{\theta}{2}}$$

Then from Coulomb's law

$$\begin{aligned} Fr^2 &= \text{a constant} \\ &= \frac{K(T+\theta)}{l \cos \frac{\theta}{2}}\ 4l^2 \sin^2 \frac{\theta}{2} \\ &= 4Kl(T+\theta) \sin \frac{\theta}{2} \tan \frac{\theta}{2}. \end{aligned}$$

Hence, by varying T we may test the truth of the law.

3. Unit of Electricity.

By Coulomb's law the force between two point charges of electricity e_1 and e_2, distant r cms. apart is given by

$$F = \frac{e_1 e_2}{r^2}.$$

Let the charges be equal and placed 1 cm. apart, then

$$F = e^2.$$

Therefore, to obtain unit force between the charges each must be a unit of charge. Hence, in the C.G.S. system, we have the following definition:

Unit charge of electricity is such that when placed in a vacuum at a distance of 1 cm. from an equal and similar charge it is repelled with a force of 1 dyne.

4. Applications of Coulomb's Law to Magnetic and to Gravitational Matter.

In magnetism, Coulomb's law may be expressed as

$$F = \frac{m_1 m_2}{r^2}$$

where m_1 and m_2 represent pole-strengths. This law may be verified by means of the torsion balance if magnetic poles are substituted for the charged pith-balls.

By a similar argument to that used above we may arrive at a definition of unit magnetic pole as being that pole-strength which will exert a force of 1 dyne on an equal and similar pole distant 1 cm. from it.

Similarly, for gravitational matter, Coulomb's law gives the relation

$$F = \frac{M_1 M_2}{r^2}$$

where M_1 and M_2 represent the respective masses of two bodies. The truth of this law may be shown by an application of Coulomb's torsion balance, which is commonly known as the Cavendish experiment, in which two small silver balls are attracted by two large lead balls. For a complete description of this experiment read the chapter on gravitation in Poynting and Thomson's "Properties of Matter".

From this gravitational law we may derive the definition of unit mass as being that mass which will exert a force of 1 dyne on an equal mass at a distance of 1 cm. This is known as the "astronomical unit of mass" and must not be confused with the ordinary units such as the gram or the pound.

5. Electric Intensity at a Point.

The electric intensity at a point is the force exerted on a unit charge placed at that point.

Theorem—The electrical intensity due to an equipotential surface is everywhere normal to the surface.

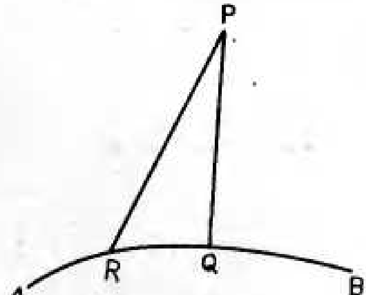

Fig 2

Let AB (Fig. 2) represent an equipotential surface; consider the electric intensity at P due to AB. Draw PQ perpendicular to AB, and let R be any point on AB.

Now the points R and Q are at the same potential, therefore, by section 14, the quantities of work required to take a unit charge along the paths PQ and PR are equal. But the path PR is equivalent to $PQ+QR$; therefore the work done in taking the charge along QR is zero. Hence the force due to AB must be normal to the surface, for if not, it would have a component which would oppose or assist the motion of the charge along QR, thereby making the work along this path not equal to zero.

A similar argument will hold for any point of AB; therefore the electrical intensity is everywhere normal to the surface.

6. Solid Angles.

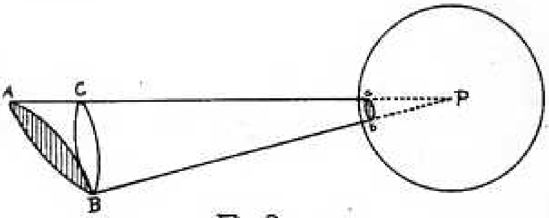

Fig 3

Let AB (Fig. 3) represent a surface, and P any point not lying on it. From every point on the boundary of AB draw lines to P, thus forming a cone with P as apex. Then the surface AB is said to subtend a solid angle at P, the angle being bounded by the cone.

In order to measure the numerical value of this angle, with P as centre draw a sphere of unit radius, on the surface o. which the cone will intercept an area ab. This area is a measure of the solid angle. If a sphere of radius PB is described about P, the cone will intercept on it an area BC, and, as will be readily seen, the surfaces AB and BC subtend the same solid angle at P. Now the areas of the surfaces ab and BC are proportional to the squares of the radii of their generating spheres, so that, if ω is the solid angle subtended at P by the surface AB,

$$\frac{\text{Area } BC}{PB^2} = \text{Area } ab = \omega$$

Where ω is small, BC may be regarded as the orthogonal projection of AB. Then if the angle between the normals to the two surfaces is θ.

$$\text{Area } BC = \text{Area } AB \times \cos\theta$$

$$\text{Therefore } \omega = \frac{\text{Area } AB \times \cos\theta}{PB^2}.$$

7. Attraction Due to a Uniform Circular Disc at a Point on its Axis.

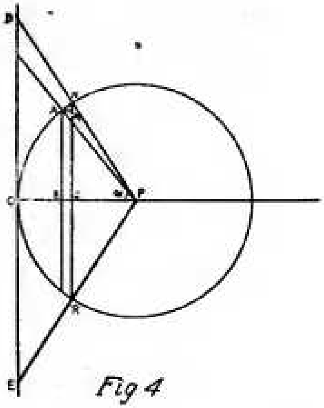

Fig 4

Let DOE (Fig. 4) represent a section through the centre of the disc, and let P be a point on the axis OP. With P as centre and PO as radius describe a sphere, and join DP and EP cutting the circular section of the sphere at N and R.

By section 6, the solid angles subtended at P by the disc DOE and the section of the sphere NOR are equal. Now the centres of the two sections are at a common distance OP from the point P, therefore it is evident that in considering the attraction at the point P we may replace the disc DOE by the section of a sphere NOR, provided that the density of charge, ρ, is the same on each.

Take now a small element AN on the circumference of the circular section and draw AB and NC perpendicular to the axis OP. If the element AN is rotated about OP, keeping the lengths AB and NC fixed, it will trace out an annular ring on the surface of the sphere The area of this elemental ring will be given by

$$\text{Area} = 2\pi AB \,.\, AN$$

$$= 2\pi AB \,.\, \frac{AM}{\sin OPA}$$

since, angle ANM = angle OPA

Now, $AB = AP \sin OPA$.

$$\therefore \text{area of ring} = 2\pi AM \,.\, AP \frac{\sin OPA}{\sin OPA}$$

$$= 2\pi AP \,.\, AM.$$

The total area of the spherical cap NOR will thus be obtained by taking successive elements of the surface from N to O, and therefore, successive values of AM from C to O.

$$i.e., \text{ total area} = 2\pi AP(OP - PB)$$
$$= 2\pi AP^2\left(\frac{OP}{AP} - \frac{PB}{AP}\right)$$
$$= 2\pi AP^2(1 - \cos\alpha)$$
since $OP = AP$,

and $\dfrac{PB}{AP} = \cos OPA = \cos\alpha$.

$$\text{Attraction at } P \text{ due to the disc } = \frac{2\pi\rho AP^2(1-\cos\alpha)}{AP^2}$$
$$= 2\pi\rho(1 - \cos\alpha).$$

If the disc be infinitely large, or if the point P is taken very close to the disc, $\alpha = 90°$, and $\cos\alpha = 0$,

$\therefore$ in this case the attraction $= 2\pi\rho$.

8. Electric Intensity at the Surface of a Charged Conductor.

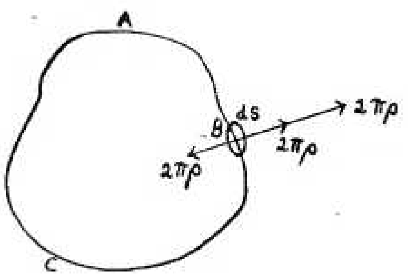

Fig. 5

Let ABC (Fig. 5) be a charged conductor of any form. At B take a circular element of area ds, so small that its surface is plane. Consider the force exerted by this little disc alone on a unit charge close to its centre. On the outside there would be a force of $2\pi\rho$ acting outwards. Similarly there would be a force of $2\pi\rho$ on the inside acting inwards. But we know that for the whole conductor there can be no force at a point inside, consequently there must be a force exerted by the rest of the conductor exactly equal and opposite to the inward force $2\pi\rho$. This is equivalent to a second outward force of $2\pi\rho$. Consequently at any point on the surface of a charged conductor,

$$R = 2\pi\rho + 2\pi\rho$$
$$= 4\pi\rho$$