FIRST PRINCIPLES OF THE DIFFERENTIAL AND INTEGRAL CALCULUS, OR THE DOCTRINE OF FLUXIONS, INTENDED AS AN INTRODUCTION TO THE PHYSICO-MATHEMATICAL SCIENCES

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ETIENNE BÉZOUT

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OF THE

DIFFERENTIAL AND INTEGRAL CALCULUS,

OR THE

DOCTRINE OF FLUXIONS,

INTENDED

AS AN INTRODUCTION TO THE PHYSICO-MATHEMATICAL SCIENCES;

TAKEN CHIEFLY

FROM THE MATHEMATICS OF BÉZOUT,

AND TRANSLATED FROM THE FRENCH

FOR THE USE OF THE STUDENTS OF THE UNIVERSITY

AT

CAMBRIDGE, NEW ENGLAND.

SECOND EDITION.

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TO THE FIRST EDITION.

THE following treatise, except the introduction and notes, is a translation of the *Principes de Calcul qui servent d'Introduction aux Sciences Physico-Mathématiques* of Bézout. It was selected on account of the plain and perspicuous manner for which the author is so well known, as also on account of its brevity and adaptation in other respects to the wants of those who have but little time to devote to such studies. The easier and more important parts are distinguished from those which are more difficult or of less frequent use, by being printed in a larger character. In the Introduction, taken from Carnot's *Reflexions sur la Metaphysique du Calcul Infinitesimal*, a few examples are given to show the truth of the infinitesimal method, independently of its technical form. Moreover in the 4th of the notes, subjoined at the end, some account is given from the same work, of the methods previously in use, analogous to the Infinitesimal Analysis. The other notes are intended to supply the deficiencies of Lacroix's Algebra (Cambridge Translation), considered as a preparatory work.

Since this treatise was announced, the compiler of the Cambridge Mathematics has been obliged, on account of absence from the country and infirmity of sight, to resign his work into other hands. This circumstance is mentioned to account for the delay attending the publication, as well as the occasional want of conformity to other parts of the course in the mode of rendering certain words and phrases which a revision of the translation, had it been practicable, would have easily remedied.

Cambridge, July, 1824.

INTRODUCTION.

THE Infinitesimal Analysis, as presented in the following Treatise, proposes to ascertain the relation of definite, assignable quantities, by comparing them with quantities which are here called *infinitely small*. But by *infinitely small* quantities is meant *quantities which may be made as small as we please, without altering the value of those with which they are compared, and whose ratio is sought*. The first idea of this calculus was probably suggested by the difficulties which are often met with in endeavouring to express by equations the different conditions of a problem, and in resolving these equations when formed. When the exact solution of a problem is too difficult, it is natural to endeavour to approximate as nearly as possible to an accurate solution, by neglecting those quantities which embarrass the combinations, if it is seen that they are so small, that the neglect of them will not materially affect the result. Thus, for example, it being found very difficult to discover directly the properties of curves, mathematicians would have recourse to the expedient of considering them as polygons of a great number of sides. For, if a regular polygon be inscribed in a circle, it is manifest, that these two figures, although they can never coincide and become the same, approach each other the more nearly in proportion as the number of the sides of the polygon increases. Whence it follows, that, by supposing the number of sides very great indeed, we may, without any very sensible error, attribute to the circle the properties which are found to belong to the inscribed polygon. And if, in the course of a calculation, we should find a circumstance in which the process would be much simplified by neglecting one of these exceedingly small sides, when compared with a radius, for example, we might evidently do it without inconvenience, since