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PAPERS READ BEFORE THE SOCIETY,

1907–1908.

I.—THE METHODS OF MODERN LOGIC AND THE CONCEPTION OF INFINITY.

By R. B. HALDANE.

IN an address delivered some two years ago and since then published, I made an attempt to estimate the influence on economic ideas of the great advances effected in the methods of modern logicians. Whether I succeeded I do not know. But the moral which I sought to point was that economists of more than one shade of opinion would do well to criticise their categories, and to see to it that they do not confuse what are in reality only valuable working conceptions with concrete realities. My purpose in this paper is to pursue the same method, and to suggest that, in yet another region, abstractions are apt to be hypostatised into realities.

Both in daily parlance and in mathematical science the word "infinite" is freely employed. Yet this expression is, as a rule, either not defined at all, or, if defined, employed in a special sense which excludes what the word means when used in other connections. An infinite series suggests, or ought to suggest, nothing analogous to an infinite God. The former may be limited by a finite quantity which the sum of the members of the series, though themselves increased in number without limit, cannot exceed. Such a numerical series can be treated as collapsing into a whole which is finite, and this whole is determined by the law of the series, through the medium of

the definite relationship between the members which constitutes that law. Finite, in the sense of ended, is in this fashion prevented from coming into contradiction with infinite, in the sense of unending. The two aspects are present in one whole, and are not inconsistent with each other. But there is nothing final about the whole which they constitute. It is one single system alongside of others, and is itself included in an indefinite succession of larger systems, which may or may not be capable of being summed in an analogous finite whole as their limit. In the region of space and time real finality is sought in vain. The progress is endless. And this is entirely true of the more general category of Quantity. Quantity as such is not a concrete thing. It is a relation or category. Even in mathematics the category of quantity is not always used. The new science of Projective Geometry seems expressly to exclude the quantitative relationships of space, and to confine itself to qualitative distinctions of points and lines. In the infinitesimal calculus the better opinion appears to be that from the notion of definite quantities or quanta, as ordinarily understood, we must free our minds and speech if we would escape self-stultification. But before I go further into this topic I should like to examine the notion of Quantity itself a little more closely.

Quantity is sometimes defined as the capacity of being increased or decreased. This definition is tautologous, but it points to a real distinction. The distinctive element in quantity is the capacity of being increased or decreased without alteration of character or quality. In pure quantity, whether we deal with occupation of space or with number, unit passes into unit, and the substratum remains through the change of quantity qualitatively identical. This is so notwithstanding that quantity possesses a discrete as well as a continuous aspect. Quantitative magnitudes are not of two kinds, but inherently possess two aspects, that of unbroken self-identity or continuity, and that of divisibility into units or parts. On whichever side

we take quantity we find ourselves faced by the capacity of indefinite extension. This capacity is sometimes spoken of as extension to infinity. But the word infinite as so used betokens no true infinite. It simply signifies that something greater or less can be set up beyond any given stage. But this something beyond is essentially existence alongside of, and in contrast to, the phase of existence already reached. It is therefore a finite. The true infinite cannot exist by contrast—cannot be a cause, even a first cause, or a substance, or numerically different from anything else; for these, all of them, carry the badge of finitude. A true infinite must be self-contained. It follows that no quantum can be infinite, and that to escape from its inherent finitude we must turn to its qualitative relations. From this side quantity may have an aspect in which it is self-contained —for instance, as the sum of a series, but it is only in one aspect that such a sum has the resemblance of self-containedness. The number 2 is the sum or limit of an arithmetical series that in point of number of members has no limit. But although in its aspect of a limit it contains the whole series, in its aspect as a definite number it has an infinity of numbers outside itself. It is therefore only *sub modo* that it is representative of what is self-contained, and for the series such a summation is possible only in virtue of a law or relationship in which the notion of a limit is inherent. From the failure of mathematicians to notice the ambiguity in the word "infinite" a great amount of apparently unnecessary controversy has arisen. The methods of modern logic, with their stringent insistence on criticism of categories, might, if carefully applied, have delivered them from much.

I am not a mathematician, and I speak with some reluctance on that which I have studied mainly as the *corpus vile* on which to attempt logical investigations. When, however, even an outsider enters on an examination of the principles of the infinitesimal calculus he is impressed with the evidences of confusion. But I will quote the language of men who are not

outsiders, two distinguished mathematicians—both Americans—who have been candid on this subject. I will begin with Dr. A. T. Bledsoe's "Philosophy of Mathematics":—

"The student of mathematics, in passing from the lower branches of the science to the infinitesimal analysis, finds himself in a strange and wholly foreign department of thought. He has not risen by easy and gradual steps, from a lower to a higher, purer, and more beautiful region of scientific truth. On the contrary, he is painfully impressed with the conviction that the continuity of the science has been broken, and its unity destroyed by the influx of principles which are as unintelligible as they are novel. He finds himself surrounded by enigmas and obscurities which only serve to perplex his understanding and darken his aspirations after knowledge."

Commenting on this passage, Professor Buckingham, of Chicago, in his striking book on the "Differential and Integral Calculus," goes further:—

"The student," he declares, "finds himself required to ignore the principles and axioms that have hitherto guided his studies and sustained his convictions, and to receive in their stead a set of notions that are utterly repugnant to all his preconceived ideas of truth. When he is told that one quantity may be added to or subtracted from another, without diminishing it; that one quantity may be infinitely small, another infinitely smaller, and another infinitely smaller still, and so on *ad infinitum*—that a quantity may be so small that it cannot be divided, and yet may contain another an indefinite and even an infinite number of times—that zero is not always nothing, but may not only be something or nothing as occasion may require, and may be *both at the same time*, in the same equation—it is not surprising that he should become bewildered and disheartened. Nevertheless, if he study the text books that are considered orthodox in this country and in Europe, he will find some of these notions set forth in them all; not indeed in their naked deformity, as they are here stated, but softened and made