LOGARITHMIC AND TRIGONOMETRIC TABLES. PREPARED UNDER THE DIRECTION OF EARLE RAYMOND HEDRICK TO ACCOMPANY THE ELEMENTS OF PLANE TRIGONOMETRY

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BY

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EXPLANATION OF THE TABLES*

TABLE I. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS FROM 1 TO 10 000

1. Powers of 10. Consider the following table of values of powers of 10:

COLUMN *A*		COLUMN *B*	COLUMN *A*		COLUMN *B*
10^1	=	10	10^0	=	1.
10^2	=	100	10^{-1}	=	.1
10^3	=	1000	10^{-2}	=	.01
10^4	=	10000	10^{-3}	=	.001
10^5	=	100000	10^{-4}	=	.0001
10^6	=	1000000	10^{-5}	=	.00001
10^7	=	10000000	10^{-6}	=	.000001
10^8	=	100000000	10^{-7}	=	.0000001
10^9	=	1000000000	10^{-8}	=	.00000001
10^{10}	=	10000000000	10^{-9}	=	.000000001

This table may be used for multiplying or dividing powers of 10, by means of the rules $10^a \cdot 10^b = 10^{a+b}$, $10^a \div 10^b = 10^{a-b}$. Thus, to multiply 1000 by 100,000, add the exponent of 10 in column *A* opposite 1000 to the exponent of 10 opposite 100,000: $3 + 5 = 8$; and look for the number in column *B* opposite 10^8, *i.e.* 100,000,000. Similarly $1,000,000 \times .0001 = 100$, since $6 + (-4) = 2$.

To divide 1,000,000 by 100, from the exponent of 10 opposite 1,000,000 subtract the exponent of 10 opposite 100; $6 - 2 = 4$; and look for the number opposite 10^4, *i.e.* 10,000. Similarly $.001 \div 1,000,000 = .000000001$, since $-3 - 6 = -9$. To find the 4th power of 100, multiply the exponent of 10 opposite 100 by 4: $4 \times 2 = 8$, and look for the number opposite 10^8, *i.e.* 100,000,000. Likewise $(.001)^3 = .000000001$, since $3 \times (-3) = -9$. To find the cube root of 1,000,000,000, divide the exponent of 10 opposite 1,000,000,000 by 3, $9 \div 3 = 3$, and look for the number opposite 10^3.

* This Explanation, written to accompany the five-place tables, may be used also for the four-place tables by omitting the last figure in each example in a manner obvious to the teacher.

2. Common Logarithms. The exponent of 10 in any row of column A is called the common logarithm * of the number opposite in column B; thus $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc.; $\log 1 = 0$, $\log .1 = -1$; $\log .01 = -2$, $\log .001 = -3$, etc. In general, if $10^l = n$, l is called the *common logarithm of* n, and is denoted by $\log n$.

3. Fundamental Principles. Logarithms are useful in reducing the labor of performing a series of operations of multiplication, division, raising to powers, extracting roots, as above; they have no necessary connection with trigonometry, since all the operations could be performed without them; but they are a great labor-saving device in arithmetical computations. They do not apply to addition and subtraction.

The principles of their application are stated as follows:

I. *The logarithm of a product is equal to the sum of the logarithms of the factors:* $\log ab = \log a + \log b$. This follows from the fact that if $10^x = a$ and $10^y = b$, $10^{x+y} = a \cdot b$. In brief: *to multiply, add logarithms.*

II. *The logarithm of a fraction is equal to the difference obtained by subtracting the logarithm of the denominator from the logarithm of the numerator:* $\log (a/b) = \log a - \log b$. For, if $10^x = a$ and $10^y = b$, then $10^{x-y} = a \div b$. In brief: *to divide, subtract logarithms.*

III. *The logarithm of a power is equal to the logarithm of the base multiplied by the exponent of the power:* $\log a^b = b \log a$. This follows from the fact that if $10^l = a$, then $10^{lb} = a^b$.

IV. *The logarithm of a root of a number is found by dividing the logarithm of the number by the index of the root:* $\log \sqrt[b]{a} = (\log a)/b$. This follows from the fact that if $10^l = a$, then $10^{l/b} = a^{1/b} = \sqrt[b]{a}$.

Corollary of II. *The logarithm of the reciprocal of a number is the negative of the logarithm of the number:* $\log (1/a) = -\log a$, since $\log 1 = 0$.

4. Characteristic and Mantissa. It is shown in algebra that every real positive number has a real common logarithm, and that if a and b are any two real positive numbers such that $a < b$, then $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

An inspection of the following table, which is a restatement of a part

a	1	10	100	1000	10000	100000	1000000	10000000
$\log a$	0	1	2	3	4	5	6	7

* Common logarithms are exponents of the base 10; other systems of logarithms have bases different from 10; Napierian logarithms (see Table VII, p. 112) have a base denoted by e, an irrational number whose value is approximately 2.71828. When it is necessary to call attention to the base, the expression $\log_{10} a$ will mean common logarithm of a; $\log_e a$ will mean the Napierian logarithm, etc.; but in this book $\log a$ denotes $\log_{10} a$ unless otherwise explicitly stated.

of the table of § 1, p. v, shows that

the logarithm of every number between 1 and 10 is a proper fraction,

the logarithm of every number between 10 and 100 is 1 + a fraction,

the logarithm of every number between 100 and 1000 is 2 + a fraction; and so on. It is evident that the logarithm of every number (not an exact power of 10) consists of a whole number + a fraction (usually written as a decimal). The whole number is called the **characteristic**; the decimal is called the **mantissa**. The characteristic of the logarithm of any number greater than 1 may be determined as follows:

RULE I. *The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.*

The following table, which is taken from § 1, p. v, shows that

a	.0000001	.000001	.00001	.0001	.001	.01	.1	1
$\log a$	-7	-6	-5	-4	-3	-2	-1	0

the logarithm of every number between .1 and 1 is -1 + a fraction,

the logarithm of every number between .01 and .1 is -2 + a fraction,

the logarithm of every number between .001 and .01 is -3 + a fraction; and so on.

Thus the characteristic of every number between 0 and 1 is a negative whole number; there is a great practical advantage, however, in computing, to write these characteristics as follows: $-1 = 9 - 10$, $-2 = 8 - 10$, $-3 = 7 - 10$, etc. *E.g.* the logarithm of .562 is $-1 + .74974$, but this should be written $9.74974 - 10$; and similarly for all numbers less than 1.

RULE II. *The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing* -10 *after the result.*

Thus, the characteristic of log 845 is 2 by Rule I; the characteristic of log 84.5 is 1 by (I); of log 8.45 is 0 by (I); of log .845 is $9 - 10$ by (II); of log .0845 is $8 - 10$ by (II).

An important consequence of what precedes is the following:

To move the decimal point in a given number one place to the right is equivalent to adding one unit to its logarithm, because this is equivalent to multiplying the given number by 10. Likewise, to move the decimal point one place to the left is equivalent to subtracting one unit from the logarithm. Hence, moving the decimal point any number of places to the right or left does not change the mantissa but only the characteristic.*

Thus, 5345, 5.345, 534.5, .05345, 534500 all have the same mantissa.

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* Another rule for finding the characteristic, based on this property, is often useful: if the decimal point were just after the first significant figure, the characteristic would be zero; start at this point and count the digits passed over to the left or right to the actual decimal point; the number obtained is the characteristic, except for sign; the sign is negative if the movement was to the left, positive if the movement was to the right.

5. Use of the Table. To use logarithms in computation we need a table arranged so as to enable us to find, with as little effort and time as possible, the logarithms of given numbers and, vice versa, to find numbers when their logarithms are known. Since the characteristics may be found by means of Rules I and II, p. vii, only mantissas are given. This is done in Table I. Most of the numbers in this table are irrational, and must be represented in the decimal system by approximations. A five-place table is one which gives the values correct to five places of decimals.

PROBLEM 1. *To find the logarithm of a given number.* First, determine the characteristic, then look in the table for the mantissa.

To find the mantissa in the table when the given number (neglecting the decimal point) consists of four, or less, digits (exclusive of ciphers at the beginning or end), look in the column marked N for the first three digits and select the column headed by the fourth digit: the mantissa will be found at the intersection of this row and this column. Thus to find the logarithm of 72050, observe first (Rule I) that the characteristic is 4. To find the mantissa, fix attention on the digits 7205; find 720 in column N, and opposite it in column 5 is the desired mantissa, .85763; hence log 72050 = 4.85763. The mantissa of .007826 is found opposite 782 in column 6 and is .89354; hence log .007826 = 7.89354 − 10.

6. Interpolation. If there are more than four significant figures in the given number, its mantissa is not printed in the table; but it can be found approximately by assuming that the mantissa varies as the number varies in the small interval not tabulated; while this assumption is not strictly correct, it is sufficiently accurate for use with this table.

Thus, to find the logarithm of 72054 we observe that log 72050 = 4.85763 and that log 72060 = 4.85769. Hence a change of 10 in the number causes a change of .00006 in the mantissa; we assume therefore that a change of 4 in the number will cause, approximately, a change of .4 × .00006 = .00002 (dropping the sixth place) in the mantissa; and we write log 72054 = 4.85763 + .00002 = 4.85765.

The difference between two successive values printed in the table is called a **tabular difference** (.00006, above). The proportional part of this difference to be added to one of the tabular values is called the **correction** (.000024, above), and is found by multiplying the tabular difference by the appropriate fraction (.4, above). These proportional parts are usually written *without the zeros*, and are printed at the right-hand side of each page, to be used when mental multiplications seem uncertain.

Example 1. Find the logarithm of .0012647. Opposite 126 in column 4 find .10175; the tabular difference is 34 (zeros dropped); .7 × 34 is given in the margin as 24; this correction added gives .10199 as the mantissa of .0012647; hence log .0012647 = 7.10199 − 10.

Example 2. Find the logarithm of 1.85643. Opposite 185 in column 6 find .26858; tabular difference 23; .43 × 23 is given in the margin as 10; this correction added gives .26868 as the mantissa of 1.85643; hence log 1.85643 = 0.26868.