

**AN ELEMENTARY TREATISE ON
THE DIFFERENTIAL CALCULUS
FOUNDED ON THE METHOD OF
RATES OR FLUXIONS**

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An Elementary Treatise on the Differential Calculus Founded on the Method of Rates Or Fluxions by John Minot Rice & William Woolsey Johnson

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JOHN MINOT RICE & WILLIAM WOOLSEY JOHNSON

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METHOD OF RATES OR FLUXIONS

BY
JOHN MINOT RICE
PROFESSOR OF MATHEMATICS IN THE UNITED STATES NAVY
AND
WILLIAM WOOLSEY JOHNSON
PROFESSOR OF MATHEMATICS IN SAINT JOHN'S COLLEGE ANNAPOLIS MARYLAND

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PREFACE.

IN preparing this abridgment of their treatise on the Differential Calculus, the authors have endeavored to adapt it to the wants of those instructors who find the larger work too extensive for the time allotted to this subject.

J. M. R.
W. W. J.

ANNAPOLIS, MARYLAND,
August, 1880.

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