MATHEMATICAL QUESTIONS WITH THEIR SOLUTIONS, FROM THE "EDUCATIONAL TIMES", VOL. XXVII. FROM JUNE TO DECEMBER 1877

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Mathematical Questions with Their Solutions, from the "Educational Times", Vol. XXVII. From June to December 1877 by W. J. C. Miller

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W. J. C. MILLER

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SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

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Pagers and Solutions not published in the "Educational Times."

EDITED BY

W. J. C. MILLER, B.A., REGISTRAR OF THE ORNERAL MEDICAL COUNCIL.

> VOL. XXVIII. FROM JUNE TO DECEMBER, 1877.

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	nuke	the expression f	27.12-123	6x+413	a rational	square	65

- 3192. (J. M. Greenwood.) The axes of two unequal cylinders intersect at right angles ; determine the surface and volume
- common to both. 3905. (Professor Wolstenholme, M.A.)—If A, B, C be three points ou a circle, there are generally four points P, P, Q, Q' on the circle such that $PA^2 = PB \cdot PC$, &c., the straight lines PP', QQ' being parallel to the external and internal bisectors of the angle A, and meeting the straight line BC in the same point as the tangent at A. The points Q, Q' will coincide when $g^2 = 4be_1$ and in this case the locus of Q (B, C being fixed points) is a rectangular hyperbola of which B, C are the foei, and the loci of P, P' are two circular cubics whose foei are B, C and one vertex of the rectangular hyperbola. These cubics are inverses of the rectangular hyperbola with respect to its

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4200. (The Editor.)-A variable square moves with its centre on the circumference of a given circle, and has one of its diagonals always parallel to a given diameter of the circle, and equal to twice the distance of the centre of the square from one end of the given diameter; show that the corners of the square move on the curves $r = 2a (\cos \theta \pm \cos 3\theta)$(1, 2), the given diameter (=2a) being the polar axis, and its end the origin. Trace these curves, showing that (1), with the upper sign, is the locus of the corners at the ends of the diagonal parallel to the given diameter; that the Cartesian equation of the curve is

 $(x^{5}+y^{2})^{2} = 4ax(x^{2}-y^{2}), \text{ or } \{y^{5}+x(2a+x)\}^{2} = 4ax^{2}(a+2x)...(3);$

that the origin is a triple point where two branches form a crunode, cutting the axis at angles of 45°, and that the third branch, near the origin, takes the form of the parabola $y^2 = -4sx$; that the curve consists of three loops, one, of length 4s and breadth 2s, to the right of the origin, and two equal loops extending to a distance is to the left of the origin :-also that (2) with the lower sime is the lower of the origin :-also that (2), with the lower sign, is the locus of the ends of the diagonal perpendicular to the fixed diameter; that the Carlesian equation of the curve is

 $(x^3+y^2)^2 = 8axy^3$, or $\{y^2-2x(4a-x)\}^2 = 8ax^2(2a-x)...(4);$

that the origin is a triple point where two of the branches form These the origin is a triple point where two of the branches form a coratoid cusp, which, near the origin, takes the form of the semi-cubical parabola $s^2 = 8ay^4$, and the third branch, near the origin, takes the form of the parabola $y^2 = 8az$; that the curve consists of two equal loops which extend to a distance of 2a to the right of the origin, and cut the large-loop branches of (1) on the circle, at an angular ordinate of 30° ; that the chord of (1) parallel to the given diameter, and the chord of (2) perpen-dicular to the given diameter, are each double the corresponding phord of the circle; and that the whole area of sect curve as chord of the circle; and that the whole area of each curve is double the area of the fixed circle.

- 4224. (Dr. Hart.)-Find three numbers such that, if the product of every two be added to the sum of the same two, the sums shall be squares: and also, if the product of every two be added to the third, the sums shall be squares. 67
- 66
- 4561. (G. Sanderson.)—A great circle of a sphere is completely surrounded by 10 tails each 2 inches in diameter; and another row of 10 equal balls lies on the sphere, each one touching two of the first row and also touching each other; find the diameter of the second-row balls.

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- No. 4743. (S. Tebay, B.A.)—Two persons A and B, whose speeds of walk-ing can vary from 0 to σ and from 0 to b miles an hour re-spectively, start from two points A, B at the same time, and Page walk uniformly in a straight line for a time $=\frac{c}{e-b}$, where

o=AB. Show that the probability that A overtakes B is

$$\frac{c}{a} \left\{ \frac{b}{2a} \cdot \frac{2a-b}{a-b} - \log \frac{a}{a-b} \right\} \dots 23$$

4830. (T. Cotterill, M.A.)—Let a function of any letters be called alternating which changes its sign only, when any two of the letters are transposed, and be denoted by the letters being placed within parentheses. If the function is of such a nature that the equation

(pxy) (pzt) + (pyz) (pxt) + (pzz) (pyt) = 0

holds for any values of letters, prove that the expression

is alternating, and including itself can be put under fifteen different forms. 26

- 4836. (Professor Wolstenholme, M.A.)—Three conics S_1, S_2, S_3 are such that the polar reciprocal of any one with respect to another is the third; a triangle ABC is inscribed in S_3 and its sides touch S_2 in a, b, c; prove that the triangle abc is self-conjugate to S_3 , and that its sides touch S_2 ; also the tangents to S_3 at A, B, C form a triangle A'B'C' which is self-conjugate to S_3 and whose angular points lie on S_3 ; also the tangent at A' passes through b, c &c. through b, c, &c..... 97
- 4915. (Dr. Collins.)-Prove that (1) the equation of the parabolic asymptote of

$$x^{n-2} x^{2} + a'y^{n-3} x^{3} + a''y^{n-3} x^{4} + &c, + by^{n-1} + b'y^{n-3} x + b''y^{n-3} x^{2} + &c, \\ + oy^{n-2} + c'y^{n-3} x + &c, = 0$$

is
$$ax^3 + by + \frac{ab' - a'b}{a}x + c + \frac{b^3(aa'' - a'^3) + ab'a'b - ab''}{a^3} = 0;$$

(2) the parabolic asymptote of third order of

ayn-8x3+a'yn-4x++a''yn-8x4+&c. + byn-1+b'yn-2x+b' yn-8x8+&c.

$$+ cy^{n-2} + c'y^{n-3}x + c''y^{n-4}x^2 + \&c. + dy^{n-3} + d'y^{n-4}x + \&c. = 0$$

is $a^3x^3 + a^5by^2 + a(ab - a'b)xy$

+
$$\{a(ab''-a''b)-a'(ab'-a'b)\}x^2 + Ay + Bx + C = 0;$$

(3) in the former case, when c is not = 0, the asymptotic com-(3) in the former case, when c is not = 0, the asymptotic common parabola above indicated has really a five-pointic contact at infinity with the given curve; and (4) find Å, B, O in the second case (which is not done by Euler in pp. 98 and 102 of his *Analysis Infinitorum*) so that the cubic asymptote partly indicated above may really have a nine-point contact at infinity with the cubic refers to a second case. with the given curve of the *n*th order when δ is not = 0; for when $\delta = 0$ in the first equation, the line projected to infinity is not a tangent to the curve, but a straight line passing through a double point of it; and when $\delta = 0$ in the second equation,

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