

LECTURES ON ELEMENTARY MATHEMATICS

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Lectures on Elementary Mathematics by Joseph Louis Lagrange

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JOSEPH LOUIS LAGRANGE

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ELEMENTARY
MATHEMATICS**



Lectures on elementary mathematics

Joseph Louis Lagrange, Thomas Joseph McCormack

LECTURES
OR
ELEMENTARY MATHEMATICS

BY
JOSEPH LOUIS LAGRANGE

TRANSLATED BY
THOMAS J. McCORMACK

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PREFACE.

THE present work, which is a translation of the *Leçons élémentaires sur les mathématiques* of Joseph Louis Lagrange, the greatest of modern analysts, and which is to be found in Volume VII. of the new edition of his collected works, consists of a series of lectures delivered in the year 1795 at the *Ecole Normale*,—an institution which was the direct outcome of the French Revolution and which gave the first impulse to modern practical ideals of education. With Lagrange, at this institution, were associated, as professors of mathematics, Monge and Laplace, and we owe to the same historical event the final form of the famous *Géométrie descriptive*, as well as a second course of lectures on arithmetic and algebra, introductory to these of Lagrange, by Laplace.

With the exception of a German translation by Niedermüller (Leipsic, 1880), the lectures of Lagrange have never been published in separate form; originally they appeared in a fragmentary shape in the *Stances des Ecoles Normales*, as they had been reported by the stenographers, and were subsequently reprinted in the journal of the Polytechnic School. From references in them to subjects afterwards to be treated it is to be inferred that a fuller development of higher algebra was intended,—an intention which the brief existence of the *Ecole Normale* defeated. With very few exceptions, we have left the expositions in their historical form, having only referred in an Appendix to a point in the early history of algebra.

The originality, elegance, and symmetrical character of these lectures have been pointed out by DeMorgan, and notably by Dühring, who places them in the front rank of elementary expositions, as an exemplar of their kind. Coming, as they do, from one of the greatest mathematicians of modern times, and with all the excellencies which such a source implies, unique in their character

as a *reading-book* in mathematics, and interwoven with historical and philosophical remarks of great helpfulness, they cannot fail to have a beneficent and stimulating influence.

The thanks of the translator of the present volume are due to Professor Henry B. Fine, of Princeton, N. J., for having read the proofs.

THOMAS J. McCORMACK.

LA SALLE, ILLINOIS, August 1, 1898.

JOSEPH LOUIS LAGRANGE.

BIOGRAPHICAL SKETCH.

A GREAT part of the progress of formal thought, where it is not hampered by outward causes, has been due to the invention of what we may call *stenographic*, or *short-mind*, symbols. These, of which all written language and scientific notations are examples, disengage the mind from the consideration of ponderous and circuitous mechanical operations and economise its energies for the performance of new and unaccomplished tasks of thought. And the advancement of those sciences has been most notable which have made the most extensive use of these short-mind symbols. Here mathematics and chemistry stand pre-eminent. The ancient Greeks, with all their mathematical endowment as a race, and even admitting that their powers were more visualistic than analytic, were yet so impeded by their lack of short-mind symbols as to have made scarcely any progress whatever in analysis. Their arithmetic was a species of geometry. They did not possess the sign for zero, and also did not make use of position as an indicator of value. Even later, when the germs of the indeterminate analysis were disseminated in Europe by Diophantus, progress ceased here in the science, doubtless from this very cause. The historical calculations of Archimedes, his approximation to the value of π , etc., owing to this lack of appropriate arithmetical and algebraical symbols, entailed enormous and incredible labors, which, if they had been avoided, would, with his genius, indubitably have led to great discoveries.

Subsequently, at the close of the Middle Ages, when the so-called Arabic figures became established throughout Europe with the symbol 0 and the principle of local value, immediate progress was made in the art of reckoning. The problems which arose gave rise to questions of increasing complexity and led up to the general solutions of equations of the third and fourth degree by the Italian mathematicians of the sixteenth century. Yet even these discoveries were made in somewhat the same manner as problems in mental arithmetic are now solved in common schools; for the present signs of plus, minus, and equality, the radical and exponential signs, and especially the systematic use of letters for denoting general quantities in algebra, had not yet become universal. The last step was definitively due to the French mathematician Vieta (1540-1603), and the mighty advancement of analysis resulting therefrom can hardly be measured or imagined. The trammels were here removed from algebraic thought, and it ever afterwards pursued its way unincumbered in development as if impelled by some intrinsic and irresistible potency. Then followed the introduction of exponents by Descartes, the representation of geometrical magnitudes by algebraical symbols, the extension of the theory of exponents to fractional and negative numbers by Wallis (1616-1703), and other symbolic artifices, which rendered the language of analysis as economic, unequivocal, and appropriate as the needs of the science appeared to demand. In the famous dispute regarding the invention of the infinitesimal calculus, while not denying and even granting for the nonce the priority of Newton in the matter, some writers have gone so far as to regard Leibnitz's introduction of the integral symbol \int as alone a sufficient substantiation of his claims to originality and independence, so far as the power of the new science was concerned.

For the *development* of science all such short-mind symbols are of paramount importance, and seem to carry within themselves the germ of a perpetual mental motion which needs no outward power for its unfoldment. Euler's well-known saying that his