THE ELEMENTS OF ANALYTICAL GEOMETRY; COMPREHENDING THE DOCTRINE OF THE CONIC SECTIONS

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The Elements of Analytical Geometry; Comprehending the Doctrine of the Conic Sections by J. R. Young & John D. Williams

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J. R. YOUNG & JOHN D. WILLIAMS

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THE

ELEMENTS

OF

ANALYTICAL GEOMETRY;

COMPREHENDING THE

DOCTRINE OF THE CONIC SECTIONS,

AND THE

GENERAL THEORY OF CURVES AND SURFACES
OF THE SECOND ORDER.

INTENDED FOR THE USE OF

MATHEMATICAL STUDENTS IN SCHOOLS AND UNIVERSITIES.

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BY J. R. YOUNG,

Author of "An Elementary Treatise on Algebra," "Elements of Geometry," &c

REVISED AND CORRECTED BY

JOHN D. WILLIAMS,

AUTHOR OF "KEY TO HUTTON'S MATHEMATICS."

PHILADELPHIA:

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TO
THE AMERICAN EDITION.

THE work now submitted to the notice of the American public, will, it is hoped, supply in part the want that has long been felt by the heads of instruction in this country, of a good elementary treatise, in our own language, upon that all important branch of mathematics—the application of analysis to the solution of Geometrical Problems. The Professors of several of our public institutions, convinced of the absolute necessity, to the student, of a thorough knowledge of this subject, prior to his entering upon the study of the Calculus, and its varied applications, have been induced to place in the hands of their pupils the works of the French writers in their native tongue. Among others, the *Essai de Geometrie Analytique* of Biot, the *Theorie des Courbes* of Boucharlat, and the *Application de l'Analyse à la Geometrie* of Bourdon, have been used to much advantage. Indeed it may be questioned if the use of the French authors as models be not almost absolutely necessary to the writer of a work on this subject; for nowhere else can we find that simple, and, at the same time, elegant and highly finished analysis, for which they are so justly distinguished in the scientific world.

Mr. Young, as will be seen by his preface, has drawn largely from these sources; and the eminent superiority of his elementary treatises on the mathematical sciences, is mainly to be attributed to the liberality of spirit with which—casting off the trammels imposed upon themselves by the countrymen of Newton—he has freely availed himself of every discovery and improvement in analysis, though such have been chiefly made on the French side of the channel.

In the present edition few alterations or additions could have been made which would improve the original, with the exception of a careful correction of the typographical errors—and whether or not the Editor has faithfully executed his task, the work itself will show.

JOHN D. WILLIAMS.

NEW YORK, August, 1833.

PREFACE.

THE application of algebra to the theory of curves and surfaces may be regarded as the fundamental branch of modern analytical science, and as the principal instrument, in conjunction with the differential and integral calculus, with which the continental mathematicians have worked such wonders in almost every department of the mathematics. The remarkable contrivance, first introduced by *Descartes*, of representing lines and surfaces by algebraical equations, enables us to embody in such an equation every property and peculiarity belonging to any curve or surface, when we know the law of its description, or any of its distinguishing characteristics; and then, to develope these several properties of the curve or surface, we have only to perform so many easy, and generally obvious, transformations on the equation which represents it. The superiority of this method over the geometrical, both in ease and fertility, immediately led to its general adoption among the French mathematicians; and the method of co-ordinates, which the Cartesian geometry involved, was afterwards applied to mechanics, and, indeed, to every other part of mathematical physics, each of which has been improved and extended by its introduction.*

English mathematicians have, however, been singularly slow in appreciating these decided advantages; so slow, indeed, that, till the year 1823, when Dr. Lardner published the first part of his Algebraic Geometry, the English language possessed not a single book on this subject. Besides this work, a treatise on analytical geometry has also emanated from the University of Cambridge, which, although a work of much originality and ability, the ingenuous author has since publicly acknowledged to be unsuited to the purposes of elementary instruction. Dr. Lardner's book will, no doubt, when completed, present a valuable body of analytical science, accessible, however, to those only who have a knowledge of the differential and integral calculus.

* Maclaurin, in his "Treatise on Fluxions," first suggested this happy idea, which threw a new light on the entire theory of mechanics. But, unfortunately, this simple principle has been neglected by later English authors, and much of what our mathematicians at present know and practise of this method, we owe chiefly to the re-importation of it through the medium of modern French works; and many, perhaps, who are admiring the facility which is thus thrown into mechanical investigations of the greatest difficulty, are unconscious that this thought had its origin in our own land.

Encyclopædia Metropolitana, art. Mechanics, p. 5.

The present little volume is then an attempt to fill up a chasm which seems still to exist in our mathematical courses of instruction, and to supply the connecting link between elementary geometry and trigonometry, and some of the most interesting applications of the transcendental analysis. For such an undertaking, the French language offers copious materials; and I have, accordingly, carefully examined and freely used the performances of Biot, Lacroix, Boucharlat, Bourdon, &c.; and I shall consider myself particularly fortunate, if it be found that I have in any degree imbibed the spirit of these elegant writers.

As regards arrangement, however, I have differed from most other authors, adopting that which appeared to me most likely to facilitate the progress of the student, without waiting to consider whether a more strictly methodical disposition of parts might not be devised. Conformably, too, to this determination, I have, in one or two cases, not hesitated to introduce a geometrical property to supply the place of analysis, where such introduction appeared of unquestionable advantage in shortening the process; instances of this occur at pages 162 and 163. The total rejection of all geometrical aid in most of French books is perhaps carried to an injudicious extent, and seems to be, in some cases, the result of caprice or affectation, for such aid is obviously allowable, and even adviseable, where simplicity may be attained by it. As to the arrangement here adopted, it may be briefly stated as follows: The volume consists of two principal parts, Analytical Geometry of Two Dimensions, and Analytical Geometry of Three Dimensions. The first part contains an introductory section on the algebraical solution of geometrical problems, and on the geometrical construction of algebraical equations; then follows, in three sections, an examination of the various properties of the lines of the second order, deduced from the most simple forms of their several equations; these three sections are, therefore, complete in themselves, comprehending, in the compass of one hundred and thirty-eight pages, a pretty copious treatise on the *Conic Sections*.

The fourth section enters more at large into the theory of these curves, by discussing very fully the most general forms of their equations, their positions in reference to any assumed axes, the determination of their varieties, &c. and the use of these researches is illustrated in Chapter iii. by their application to a variety of interesting problems on geometric loci. This first part terminates with a supplementary chapter containing some very useful theorems, such, for instance, are those at pages 211 and 214, the former of which is necessary in one very elegant mode of establishing the fundamental problem of physical astronomy, viz. that the planets move in elliptic orbits, having the sun in one of their foci; and the other problem is the foundation of the method of interpolations, so useful in the construction of tables, and in practical astronomy.

The second part is devoted to the consideration of lines and surfaces in space, the developement of their properties, and the general discussion of their equations. As to the first part, so here, a supplementary chapter is appended, containing many curious and interesting applications of the preceding theory. Most of the problems in this chapter have appeared before, some in the Annales Mathématiques, others in Leybourn's Repository, &c. but the solutions here given are for the most part new, and I think improved.

By way of index to the various topics embraced in the work, I have prefixed to the volume a very copious table of contents, which indeed precludes the necessity of extending further these prefatory remarks. I therefore conclude with the hope that the little volume now submitted to notice, though its pretensions be as humble as the form which it has assumed, may yet prove of some service to the mathematical student, in the earlier stages of his progress.

J. R. YOUNG.

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