OPTICAL INVESTIGATIONS. CAUSTICS

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PREFACE.

THE Caustic of any proposed reflecting or refracting curve is the curve formed by the perpetual intersections of rays which have been reflected or refracted at the curve. Optically considered the determination of the equations of the caustics of the curves is more important than is commonly imagined, and is closely connected with the subject of optical images. If a luminous point be situated any where in the plane of a reflecting or refracting arc, and if an eye, situated in this plane, receive one of the reflected or refracted rays, it is sufficiently apparent that the luminous point will appear to the eye just mentioned to be situated somewhere in the direction of the ray which it receives. A question arises at what particular point of this indefinite line does the visible image of the luminous point appear, and in what way is this point to be determined for any given position of the luminous point, the eye, and the reflecting arc? This is a question to which theory cannot supply any answer. It is a result of *experiment* that the eye, looking as above described, along one of the reflected or refracted rays, will see the visible image of the luminous point at the particular point in which the said ray is a tangent to the caustic touched by every one of the reflected pencils. Thus, to take a very simple instance, if the luminous point be situated at the extremity of the diameter of a reflecting circle, it will be shewn that the caustic touched, in this case, by the reflected rays, is a cardioide; and this result, viewed with reference to its optical importance, exhibits to us as in a picture the varying position of the visible image of the luminous point corresponding to the varied position of the eye of the observer. From any given position of the eye, if we draw a tangent to the curve of the caustic we in fact, in constructing the point of contact, construct for the place of the visible image of the luminous point corresponding to the proposed situation of the eye.

Again, if a luminous point be placed before a plane refracting surface of a denser medium, it may be shewn that the caustic curve is the evolute of an ellipse; and the position and magnitude of the curve may be determined for any given position of the luminous point and the refracting surface.

If the eye of an observer be supposed to be situated in the plane of the caustic, and to receive one of the refracted rays, it follows, from the experimental principle before described, that the point in which the ray just mentioned touches the caustic curve, is the place of the visible image of the luminous point corresponding to the proposed place of the eye.

If in any proposed case, while the eye of the observer, and the position of the reflecting or refracting surface remain unchanged, the place of the luminous point is varied, it easily follows that the caustic curve will in general vary in position and magnitude, whilst its species is unaltered. At the same time, the place of the point which is the visible image of the luminous point, that is to say, the point in which a reflected or refracted ray, passing through the eye, touches the caustic, will in general be changed. And further if, on the same suppositions, the luminous point move in any given curve, it is plain that the said point of contact, and therefore the place of the visible image of the luminous point, will move in a certain curve. And the determination of the curve last mentioned, for any given form of the luminous curve, is, in fact, the problem of optical images in its most general form. The explanation of the mode of effecting the solution of this problem, would be foreign to the subject of the following pages; and the problem itself has been mentioned merely for the purpose of instancing the *optical* application of the forms of caustics.

It is the object of the present publication to determine, by a simple and satisfactory method, the equations of a variety of these curves; and it is believed that, independently of their physical applications, the simplicity and analytical elegance of some of the results, may excuse the space devoted to the examination of them.

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§. 1. In Fig. 1. Q is a luminous point from which proceed rays QR, QS, . . . that fall upon the concave reflecting arc $RSTU$ . . and are reflected into the directions Rq, Sr, Ts, respectively, intersecting each other in the points q, r, s, . . The curve which passes through the points q, r, s, . . when these points approach each other indefinitely, that is to say, the curve formed by the perpetual intersections of the reflected rays, is termed a caustic.

§. 2. In order to find the equation of this curve referred to rectangular coordinates, let us investigate a method of determining, in general, the nature of the curve formed by the perpetual intersections of right lines, any one of which is defined by the equation

$$y = ax + \phi(a) \qquad (1)$$

in which $\phi(a)$ denotes any function of (a).

If we consider a second line of the system for which (a) becomes any other quantity

$$a + \delta a$$

it is plain that the equation of this line will be

$$y = \left\{a + \delta a\right\} . x + \phi\left\{a + \delta a\right\} \qquad (2)$$