SUPPLEMENTARY ALGEBRA

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Supplementary algebra by R. L. Short

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PREFACE

The large number of requests coming from teachers for supplementary work in algebra, especially such work as cannot be profitably introduced into a text, has led me to collect such material into a monograph, hoping by this means to furnish the teacher with methods and supplementary work by which he may brighten up the algebra review.

As far as possible, illustrations have been drawn directly from calculus and mechanics, — this being especially true in the problems for reduction. Almost without exception, such algebraic forms are common to calculus work. For a large part of this list of algebraic forms in calculus, I am indebted to Miss Marion B. White, Instructor in Mathematics in the University of Illinois.

No attempt has been made to demonstrate the theory hinted at in graphical work. Such treatment would involve a knowledge of higher mathematics.

The graph of the growth of state universities was made by members of a freshman class of the University of Illinois.

R. L. S.

SUPPLEMENTARY ALGEBRA.

Graphs.

1. It is impossible to locate absolutely a point in a plane. All measurements are purely relative, and all positions in a plane or in space are likewise relative. Since a plane is infinite in length and infinite in breadth, it is necessary to have some fixed form from which one can take measurements. For this form, assumed fixed in a plane, Descartes (1596–1650) chose two intersecting lines as a coördinate system. Such a system of coördinates has since his time been called Cartesian. It will best suit our purpose to choose lines intersecting at right angles.

2. The Point. If we take any point M, its position is determined by the length of the lines $QM = x$ and $PM = y$, the directions of which are parallel to the intersecting lines OX and OY (Fig. 1). The values $x = a$ and $y = b$ will thus determine a point. The unit of length can be arbitrarily chosen, but when once fixed remains the same throughout the problem under discussion. $QM = x$ and $PM = y$, we call the *coördinates* of the point M. x, measured parallel to the line OX, is called the *abscissa*. y, measured parallel to the line OY, is the *ordinate*. OX and OY are the *coördinate axes*. OX is the axis of x, also called the axis of abscissas. OY is the axis of y, also called the axis of ordinates. O, the point of intersection, is called the *origin*.

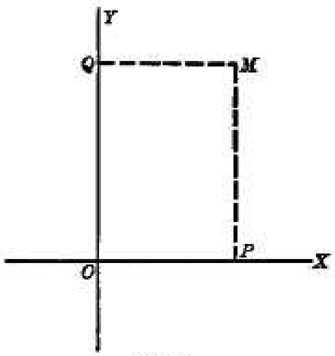

FIG. 1.

A plane has an infinity of points in its length, also an infinity of points in its breadth. The number of points in a plane being thus, ∞^2, or twofold infinite, two measurements are necessary to locate a point in a plane.

For example, $x = 2$ holds for any point on the line AB (Fig. 2). But if in addition we demand that $y = 3$, the point is fully determined by the intersection of the lines AB and CD, any point on CD satisfying the equation $y = 3$.

3. The Line. Examine one of the simplest conditions in x and y, for example $x + y = 6$. In this equation, when values are assigned to x, we get a

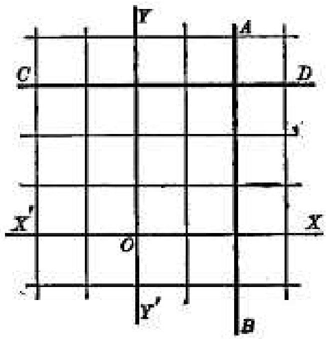

FIG. 2.

value of y for every such value of x. When $x = 0$, $y = 6$; $x = 1$, $y = 5$; $x = 2$, $y = 4$; $x = 3$, $y = 3$; $x = 5$, $y = 1$; etc., giving an infinite number of values of x and y which satisfy the equation. There is, then, no definite solution.

Laying off these values on a pair of axes, as shown in paragraph 2, we see that the points satisfying this equation lie on the line AB (Fig. 3). It is readily seen that there might be confusion as to the direction from the origin in which the measurements should be taken. This is avoided by a simple convention in signs. Negative values of x are measured to the left of the y-axis, positive to the right. In like manner, negative values of y are measured downward from the x-axis, positive values upward. The regions XOY, YOX', $X'OY'$, $Y'OX$, are spoken of as the first, second, third, and fourth quadrants respectively. (See Fig. 2.)

By plotting other equations of the first degree in two variables (two unknown quantities), it will be seen that such an equation always represents a

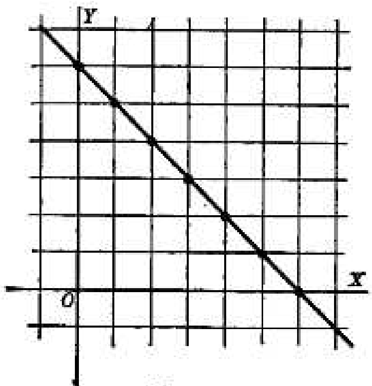

FIG. 3.

straight line. This line AB (Fig. 3) is called the graph of $x + y = 6$ and is the locus of all the points satisfying that equation.

4. Now plot two simultaneous equations of the first degree on the same axes, *e.g.* $x + y = 6$ and $2x - 3y = -3$ (Fig. 4), and we see at once that the coördinates of the point of intersection have the same values as the x and y of the algebraic solution of the equations.

This is, then, a geometric or graphical reason why there is but one solution to a pair of simultaneous equations of the first degree in two unknowns. A simple algebraic proof will be given in the next