GENERAL THEORY OF THE LAMBERT CONFORMAL CONIC PROJECTION: CARTOGRAPHY

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General Theory of the Lambert Conformal Conic Projection: Cartography by Oscar S. Adams

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DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, SUPERINTENDENT

CARTOGRAPHY

GENERAL THEORY OF THE LAMBERT CONFORMAL CONIC PROJECTION

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PREFACE.

This publication gives a general development of the theory of the Lambert conformal conic projection. It is intended to supplement the matter found in Special Publication No. 47 entitled, "The Lambert Conformal Conic Projection with Two Standard Parallels." It is also supplementary in a way to Special Publication No. 49, which contains the Lambert projection tables for the region in France, and to Special Publication No. 52, which gives corresponding tables for the United States, since it gives as a whole the mathematical development of the theory upon which they depend.

A short account of Lambert's life and work is given in the introductory paragraphs, followed by a few pages upon the subject of projections in general.

ILLUSTRATIONS.

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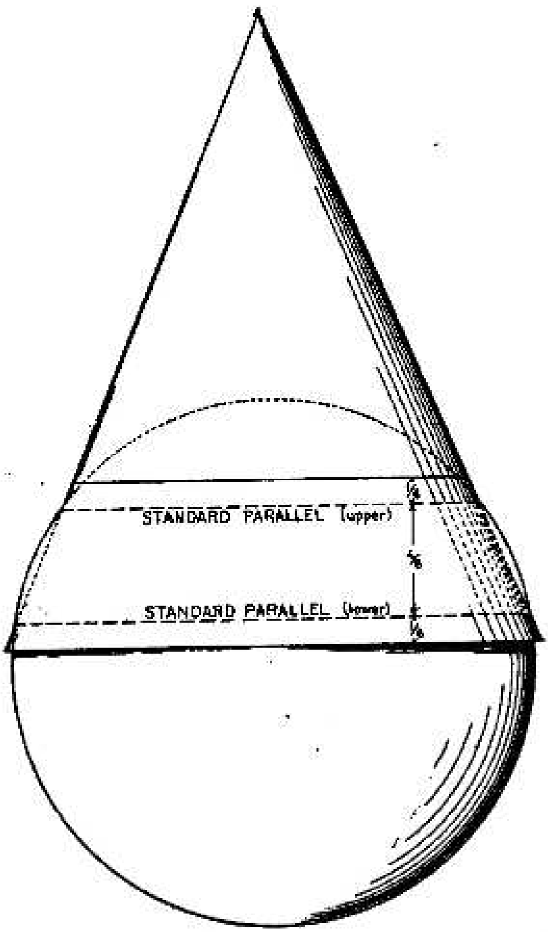

FRONTISPIECE.—Intersection of a cone and sphere along two standard parallels.

GENERAL THEORY OF THE LAMBERT CONFORMAL CONIC PROJECTION.

By Oscar S. Adams,
Geodetic Computer, U. S. Coast and Geodetic Survey.

Since this publication is to treat of some of Lambert's work, it is altogether fitting that it should be prefaced with a short account of his life, especially with a statement of his significance in the domain of map projections. Johann Heinrich Lambert was born at Mülhausen in Alsace in 1728. He was the son of a poor tailor and his education was entirely the product of his own exertions, due to a systematic course of reading. It was his regular custom to spend 17 hours per day in study and writing. At the early age of 16 he discovered, in computations for the comet of 1744, the so-called Lambert's theorem. During the latter part of his life he resided in Berlin, where he was much honored for his ability. It was in the application of mathematical analysis to the practical problems of life that he especially excelled. His untimely death occurred in 1777 in the forty-ninth year of his age.

His contributions to mathematics were the series which bears his name, the conception of hyperbolic functions, his theorem in conics, and the demonstration of the incommensurability of π. Both Lagrange and Gauss used parts of his work as points of departure for their investigations.

Lambert's work in the field of map projections needs careful consideration. He was the first mathematician to make general investigations upon the subject of projections. Those who preceded him in this work limited themselves to the development of a single method of projection, principally the perspective, but Lambert considered the problem of the representation of a sphere upon a plane from a higher standpoint and he stated certain general conditions that the representation was to fulfil, the most important of these being the preservation of angles or conformality and equal surface or equivalence. These two qualities, of course, can not be obtained in the same projection.

Although Lambert did not fully develop the theory of these two methods of projection, yet he was the first to express clearly the ideas regarding them. The former, conformality, has become of the greatest importance to pure mathematics, but both of them are of exceeding importance to the cartographer. It is no more than just, therefore, to date the beginning of a new epoch in the science of map making from the appearance of Lambert's work. What he accomplished is of importance, not only for the generality of the ideas underlying it, but also for his successful application of them in methods of projection. The manner in which he attacks and solves any particular problem is very instructive. He has developed several methods of projection that are not only interesting but that are to-day in use among cartographers, the most important of these being the conformal conic projection.

The initial problem presented to us in map making is the representation upon a plane surface of the relative positions, sizes, and shapes of features that are found upon the curved surface of the earth. A perfect representation is impossible, since the surface of the earth being non-developable, can not be spread out in a plane. There are, however, many different ways of obtaining approximate representations, the theory and properties of which constitute the subject of map projections.

The positions of the points upon the earth are usually defined by their latitude and longitude. Hence, if we can devise a suitable method of representing the meridians and parallels upon the sheet, the points can be plotted by their positions relative to these lines and the map can be constructed. The term "projection" is evidently used in a wider sense than that which is given to it in geometry. The majority of map projections are not projections in the geometrical sense—that is, perspective projections, orthogonal projections, etc.—but merely a network of meridians and parallels that makes possible a one-to-one correspondence between the places upon the earth and the points upon the map.

Some of the things to be desired in a map are as follows:

1. Preservation of the shapes of the countries.

2. Preservation of the relative sizes of the countries in their representation upon the map.

3. The distance between places should be in constant ratio to their distances as indicated upon the map.

4. A great circle upon the earth should be represented by a straight line upon the map.

5. The latitude and longitude of any place should be readily found from its position upon the map.

6. The ease with which a projection can be constructed is also to be considered from the practical standpoint.

Only part of these things can be attained by any given method of projection.

The scale of a map in any given direction is the ratio which a short distance measured upon the map bears to the corresponding distance upon the surface of the earth. The definition is limited to short distances because the scale of a map generally varies from point to point. It would of course be desirable that the scale of the map should be correct in every direction at every point and constant for all parts of the map. This is impossible, however, since if it were true, the map would be a perfect representation of the spheroidal surface and could be fitted to it. In any given method of projection, therefore, some of the features to be desired must be sacrificed.

The representation of the shapes of countries as nearly correct as possible is one of the most important functions of a map. A large country can not, of course, be represented without some distortion when considered as a whole, but small areas may be mapped by similar figures. A projection that preserves the similarity of small areas is called orthomorphic. The representation of one surface upon another so as to preserve similarity of elements has been called by mathematicians conformal representation. An orthomorphic projection is therefore a conformal representation of the spheroidal surface of the earth upon a plane. Orthomorphic projections are in general not of