

**MATHEMATICAL QUESTIONS, WITH THEIR
SOLUTIONS, FROM THE "EDUCATIONAL
TIMES", WITH MANY PAPERS
AND SOLUTIONS NOT PUBLISHED IN THE "
EDUCATIONAL TIMES". VOL. XV. FROM
JANUARY TO JUNE, 1871**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649449897

Mathematical Questions, with Their Solutions, from the "Educational Times", with Many Papers and Solutions not Published in the " Educational Times". Vol. XV. From January to June, 1871 by W. J. C. Miller

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

W. J. C. MILLER

**MATHEMATICAL QUESTIONS, WITH THEIR
SOLUTIONS, FROM THE "EDUCATIONAL
TIMES", WITH MANY PAPERS
AND SOLUTIONS NOT PUBLISHED IN THE "
EDUCATIONAL TIMES". VOL.
XV. FROM JANUARY TO JUNE, 1871**

MATHEMATICAL QUESTIONS,

WITH THEIR

SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

WITH MANY

Papers and Solutions not published in the "Educational Times."

EDITED BY

W. J. C. MILLER, B.A.,

VICE-PRINCIPAL OF HUDDESFIELD COLLEGE.

VOL. XV.

FROM JANUARY TO JUNE, 1871.

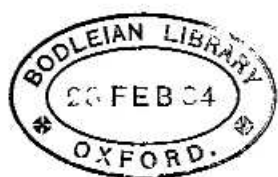
LONDON:

C. F. HODGSON & SON, GOUGH SQUARE,

FLEET STREET.

1871.

18753 e 2.



LIST OF CONTRIBUTORS.

- ALEXANDER, P., Gerton, Manchester.
 ANDERSON, D. M., Kirriemuir, Scotland.
 BALL, Professor ROBT. STAWELL, M.A., Dublin.
 BATTAGLINI, GIUSEPPE, Professor of Mathematics in the University of Naples.
 BIRKMAN, J. S., Brompton Street, Gloucester.
 BILLS, SAMUEL, Newark-on-Trent.
 BLAKEMORE, J. W. T., B.A., Stafford.
 BLISSARD, Rev. J., B.A., The Vicarage, Hampstead Norris, Berks.
 BOOTH, Rev. Dr., F.R.S., The Vicarage, Stone, Bucks.
 BOUENS, A. A., Atherston.
 BOWDITCH, W. L., Wakefield.
 BRANQUART, PAUL, Church House, Ealing.
 BRONFIELD, S. W., Christ Church College, Oxford.
 BROWN, A. CUM, D.Sc., Edinburgh.
 BURNHURD, W. S., M.A., Trinity College, Dublin.
 CARE, G. S., M.A., Woolwich Common.
 CAREY, JOHN, B.A., Kingstown, Ireland.
 CAYLEY, A., F.R.S., Sadlerian Professor of Mathematics in the University of Cambridge; Corresponding Member of the Institute of France.
 CHADWICK, W., Oxford.
 CLARKE, Captain A. R., R.E., F.R.S., Ordnance Survey Office, Southampton.
 CLIFFORD, W. K., M.A., Fellow of Trinity College, Cambridge.
 COCKLE, Sir J., M.A., F.R.S., Chief Justice of Queensland; President of the Queensland Philosophical Society.
 COHEN, ARTHUR, M.A., London.
 COLLINS, MATTHEW, B.A., Eden Quay, Dublin.
 COLQUHOUN, MILLICENT, Inverness.
 CONWILL, J., Leighlinbridge, Ireland.
 CONOLLY, E., Mountmugent, Ireland.
 COTTEHILL, THOS., M.A., London, late Fellow of St. John's College, Cambridge.
 CREMONA, LUIGI, Professor, R. Istituto Tecnico Superiore di Milano.
 CROFTON, M. W., B.A., F.R.S., Prof. of Math. in the Royal Military Acad., Woolwich.
 DALE, JAMES, Kingsland Place, Aberdeen.
 DAVIS, WILLIAM HARRITT, B.A., London.
 DEBSON, T., B.A., Head Master of Hestham Grammar School.
 DUPAIN, J. C., Professeur au Lycée d'Angoulême.
 EASTHERY, W. B.A., Grammar School, St. Asaph.
 EVANS, ASHER B., M.A., Lockport, New York, United States.
 EVERETT, J. D., D.C.L., Professor of Natural Philosophy in the Queen's University, Belfast.
 FENWICK, STEPHEN, F.R.S., Mathematical Master in the E. M. Acad., Woolwich.
 FERBER, Rev. N. M., M.A., Cains College, Cambridge.
 FICKLIN, J. W., Professor of Mathematics and Natural Philosophy, University of Missouri, United States.
 FITZGERALD, E., Bagenalstown, Ireland.
 FLOOD, P. W., Ballingarry, Ireland.
 GARDNER, MARTIN, late Professor of Mathematics in St. John's College, Sydney.
 GEESSE, R. W., B.A., Mathematical Master in the Grammar School, Manchester.
 GEXEX, E. W., B.A., Grammar School, Manchester.
 GERAGHTY, W., Blackhall Street, Dublin.
 GLAISHER, J. W. L., B.A., Trinity College, Cambridge.
 GODFREY, HUGH, M.A., Cambridge.
 GODWARD, WILLIAM, Law Life Office, London.
 GRANT, S. G. W., Royal Naval School, New Cross.
 GREAVES, MYLA E., Rue de la Paix, Boulogne-sur-Mer, France.
 GREENWOOD, JAMES M., Kirksville, Missouri, United States.
 GREER, H. R., B.A., Mathematical Master in the E. M. (Cadets') Coll., Sandhurst.
 GRIFFITH, W., Superintendent of Public Schools, Fairfield, Ohio, United States.
 GRIFFITHS, J., M.A., Fellow of Jesus College, Oxford.
 HALL, H. S., M.A., Clifton College.
 HANLON, G. O., Dublin.
 HANNA, W., Literary Institute, Belfast.
 HARLEY, Rev. ROBERT, F.R.S., Leicester.
 HART, Dr. DAVID S., Stonington, Connecticut, United States.
 HERMITE, Ch., Membre de l'Institut, Paris.
 HILL, Rev. E., M.A., St. John's College, Cambridge.
 HIRST, Dr. T. A., F.R.S., Registrar of the University of London.
 HOPKINSON, J., D. Sc., B.A., Oxford Road, Manchester.
 HOPKINS, Rev. G. H., M.A., Shirwell Road, Clonsilla, Birkenhead.
 HOPPS, WILLIAM, Leonard Street, Hull.
 HOSKINS, HENRY, Alma Place, Kensington.
 HUDSON, C. T., LL.D., Manila Hall, Clifton.
 HUDSON, W. H. H., M.A., Fellow of St. John's College, Cambridge.
 INGLEY, C. M., M.A., LL.D., Royal Society of Literature, St. Martin's Place, London.
 JENKINS, MORGAN, M.A., late Scholar of Christ's College, Cambridge, Westbourne Park, London.
 KIRKMAN, Rev. T. P., M.A., F.R.S., Croft Rectory, near Warrington.
 KITCHENER, F. E., M.A., Rugby School.
 KITCHIN, Rev. J. L., M.A., Head Master of Bideford Grammar School.

- KNISELY**, Rev. U. J., Newcomerstown, Ohio, United States.
LAVERTY, W. H., M.A., Queen's College, Oxford.
LAW, C., Cambridge.
LEVY, W. H., Shalbourne, Berks.
MADDERN, W. M., Trinity Parsonage, Wakefield.
MANSHEIM, M., Professeur à l'École Polytechnique, Paris.
MARTIN, ARTEMAS, Mathematical Editor of "The Visitor," McKean, Erie Co., Pennsylvania, United States.
MARTIN, Rev. H., M.A., Examiner in Mathematics and Natural Philosophy in the University of Edinburgh.
MASON, J., East Castle Colliery, near Newcastle-on-Tyne.
MATHEWS, F. C., M.A., London.
MATTHESON, Dr. JAMES, De Kalb Centre, Illinois, United States.
MCCATY, W. B., B.A., Trinity College, Dublin.
MCCOLL, HUGH, Rue du Temple, Boulogne-sur-Mer, France.
MCCORMICK, E., Ledbury, Hereford.
MCDOWELL, J., M.A., F.R.A.S., Pembroke College, Cambridge.
MCLEAM, ABRAHAM, New Lisbon, Ohio, United States.
MCNEILL, JAMES A., Belfast.
MERRIFIELD, C. W., F.R.S., Principal of the Royal School of Naval Architecture, South Kensington.
MERRIFIELD, J., Ph.D., F.R.A.S., Science School, Plymouth.
MILLER, W. J. C., B.A., Park Villa, Huddersfield.
MINCHIN, G. M., B.A., Trinity College, Dublin.
MITCHERSON, T., F.R.I.S., The Cedars, Rickmansworth.
MOFFITT, J., Nebblesworth, near Durham.
MOON, ROBERT, M.A., late Fellow of Queen's College, Cambridge, Cleveland Square, London.
MORLEY, THOS. I. C. P., Bromley, Kent.
MOULTON, J. F., M.A., Fellow of Christ's College, Cambridge.
MURPHY, HUGH, Pembroke Road, Dublin.
NELSON, H. J., M.A., Sailor's Institute, Naval School, London.
O'CAVANAGH, PATRICK, Dublin.
O'GILTYE, G. A., Leiston, Saxmundham.
OTTER, W. CURTIS, F.R.A.S., Liverpool.
PANTON, A. W., B.A., Trinity College, Dublin.
POLLIGNAC, Prince Camille de, Rue de Berry, Paris.
RENSHAW, S. A., Elm Avenue, New Basford, Nottingham.
REODER, HELEN, Helensborough.
RIPPIN, CHARLES E., M.A., Woolwich Common.
ROBERTS, SAMUEL, M.A., Holloway, London.
ROBERTS, Rev. W., M.A., Fellow and Senior Tutor, Trinity College, Dublin.
ROBERTS, W., Junior, Trinity College, Dublin.
ROCKER, A. W., B.A., Brasenose College, Oxford.
RUTHERFORD, Dr., F.R.A.S., Woolwich.
SALMON, Rev. G., D.D., F.R.S., Fellow of Trinity College, Dublin.
SANDERS, J. B., Bloomington, Indiana, United States.
SANDERSON, Rev. T. J., M.A., Littleton Vicarage, Boyston.
- SAVAGE**, THOMAS, M.A., Fellow of Pembroke College, Cambridge; Mathematical Master in the Royal Military (Staff) College, Sandhurst.
SCOTT, JUSTIN, Judge of the Ohio Supreme Court, Bucyrus, United States.
SHARPE, Rev. H. T., M.A., Vicar of Cherry Marham, Norfolk.
SIDES, J. J., Rue des Vieillards, Boulogne-sur-Mer, France.
SIVERLY, WALTER, Oil City, Pennsylvania, United States.
SPOTTISWOODE, WILLIAM, M.A., F.R.S., Grosvenor Place, London.
SPRAGUE, THOMAS BORD, M.A., London.
STANLEY, ARCHER, London.
SWAINSON, T., Cleator, Whitehaven.
SYMES, R. W., B.A., London.
STYVESSEE, J. J., LL.D., F.R.S., Corresponding Member of the Institute of France.
TAIL, P. G., M.A., Professor of Natural Philosophy in the University of Edinburgh.
TARLETON, FRANCIS A., M.A., Fellow of Trinity College, Dublin.
TAYLOR, C., M.A., Fellow of St. John's College, Cambridge.
TAYLOR, H. M., B.A., Fellow of Trinity College, Cambridge; Vice-Principal of Royal School of Naval Architecture, South Kensington.
TAYLOR, J. H., B.A., Cambridge.
TEBAY, SEPTIMIUS, B.A., Head Master of Rivington Grammar School.
THOMSON, F. D., M.A., Grammar School, Ezelet.
TODDUSTE, ISAAC, F.R.S., St. John's College, Cambridge.
TOMLINSON, H., Christ Church College, Oxford.
TORBELL, GABRIEL, Naples.
TORREY, Rev. A. F., M.A., St. John's Coll., Camb.
TOWNSEND, Rev. E., M.A., F.R.S., Fellow of Trinity College, Dublin.
TUCKER, R., M.A., Mathematical Master in University College School, London.
TURBELL, I. H., Cumminsville, Ohio, United States.
VOSE, G. B., Professor of Mechanics and Civil Engineering, Washington and Jefferson College, Washington, United States.
WALKER, J. J., M.A., Vice-Principal of University Hall, London.
WALMSLEY, J., B.A., Eccles, Manchester.
WARREN, E., M.A., Trinity College, Dublin.
WATSON, STEPHEN, Haydonbridge, Northumberland.
WHITE, Rev. J., M.A., Brook Hill Park, Plumstead.
WHITWORTH, Rev. W. A., M.A., Fellow of St. John's College, Cambridge.
WILKINSON, Rev. M. M. U., Reepham Rectory, Norwich.
WILKINSON, T. T., F.R.A.S., Grammar School, Rurnhey.
WILLIAMSON, B., M.A., Fellow of Trinity College, Dublin.
WILSON, JAMES, Cork.
WILSON, Rev. E., D.D., Chelsea.
WOLSTENHOLME, Professor, M.A., Cooper's Hill College, Staines.
WOODCOCK, W. S. B., F.R.A.S., &c., Alwyne Lodge, Canonbury, London.
YOUNG, J. R., Priory Cottage, Peckham.

Contributors deceased since the Publication of Vol. I.

- DE MORGAN**, AUGUSTUS, F.R.A.S.; **DE MORGAN**, G. C., M.A.; **HOLDITCH**, Rev. H., M.A.; **LRA**, W.; **O'CALLAGHAN**, J.; **FURKISS**, H. J., B.A.; **FRONNEY**, E.; **SADLER**, G. T., F.R.A.S.; **WRIGHT**, Rev. R. H., M.A.

CONTENTS.

Mathematical Papers, &c.	
No.	Page
76. Probability Notation. By HUGH M'COLL	20
77. A Solution of Newton's Theorem by the Method of Tangential Coordinates. By the Rev. Dr. BOOTH, F.R.S.	49
78. Note on Maxima and Minima. By ARTEMAS MARTIN	109
79. Extraction of the Roots of whole Numbers by the Binomial Theorem. By ARTEMAS MARTIN	110

Solved Questions.

1100. A straight line is divided at random into three segments; find the probability that it may be possible to form, with these segments, an acute-angled triangle	35
1950. If ABCD be a quadrilateral inscribed in a circle, and the sides produced meet at F and G, prove that the bisectors of the angles at F and G meet at right angles	96
2319. If O, O' be the centres of two circles, prove that the polar of any point on (O) with reference to (O') envelopes a conic of which O' is a focus	30
2401. Show that $\Delta^n 0^{n+r} = [(x_1^n n)^r (1)]_n$; that is, the r th operation of Σ_1^n upon 1, not of Σ_1^n upon n ; and that $(x_1^n n)^r (1) = n^{[r+1]} \phi_r(n)$, where $n^{[r+1]}$ denotes $n(n+1) \dots (n+r)$, and $\phi_r(n)$ is an algebraic function of n of the $(r-1)$ th degree with integral or fractional coefficients	68
2516. In the sides BC, CA, AB of a triangle ABC, points D, E, F are taken such that BD : CD = CE : AE = AF : BF = l : m. Find the average area (1) of the triangle (Δ_1) whose sides are equal to the lines AD, BE, CF, and (2) of the triangle (Δ_2) whose vertices are the intersections P, Q, R of the lines AD, BE, CF. Find also the respective probabilities (p_1, p_2) that, if D be taken at random in BC, (3) the triangle (Δ_1), or (4) the triangle (Δ_2), will be less than the average area	47

No.	Page
2656. With a given major axis all possible ellipses are described; show that for a given abscissa the centres of curvature lie on a certain cubic. Find the envelope of this for different values of the abscissa	45
2752. Through a point O, within a triangle ABC, parallels DE, FG, HI are drawn to BC, CA, AB respectively; find the locus of O when one of the three triangles AGH, BDI, CFE, external to the hexagon GDIFEH, is (1) equal to the sum of the other two, or (2, 3, 4) an arithmetic, geometric, or harmonic mean between them	96
2833. 1. Through every point A on a central conic pass three circles, which osculate the circle elsewhere, say in B, C, D. Prove that the diameter of the circle passing through ABCD is $= \frac{a^2 + 3b^2}{2b}$ when greatest, and $= \frac{b^2 + 3a^2}{2a}$ when least; and when this circle is given in magnitude, show how to find (by rule and compass) its position with respect to the conic. 2. Prove also that the distance between the centre of the circle through ABCD and the centre of the circle osculating the conic at B, (C or D,) is when greatest $= \frac{3}{4} \frac{a^2 - b^2}{b}$, and when least $= \frac{3}{4} \frac{a^2 - b^2}{a}$, where a and b are, as usual, the semi-axes of the conic	29
2899. Find (1) the average area of all the triangles that can be formed by joining three points taken at random in the circumference of a given circle, and (2) the average area of all the quadrilaterals that can be formed by joining four points taken at random in the circumference of the same circle.....	107
2963. A cone filled with fluid is placed with a generating line horizontal, and the axis in a vertical plane above it; and a small hole is made at the lowest point of the base. Find the time of emptying, and the mean value of the latus rectum of the issuing jet	53
3011. Two points are taken on a parabola such that the normals intersect on the evolute; then (1) the chord through them envelopes a parabola passing through the vertex; (2) the envelope of the circle through the points and the vertex is a quartic curve passing through the vertex	98
3015. A point is taken at random in the surface of a circle, and a chord drawn through it; find (1) the average length of the chord, and (2) the average area of the segment cut off by the chord	65
3024. Find integral values of m, n, r, s that will satisfy the condition $m^2 - n^4 = 5(m^2 - r^2) = 7(m^2 - s^2)$	106
3044. Planes passing through an axis of an ellipsoid cut it in sections whose real foci lie in the plane of the other axes. Find the locus of the foci, which is of the fourth degree.....	28