

**AN ELEMENTARY TREATISE
ON THE INTEGRAL CALCULUS:
FOUNDED ON THE METHOD
OF RATES OR FLUXIONS**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649053896

An Elementary Treatise on the Integral Calculus: Founded on the Method of Rates Or Fluxions
by William Woolsey Johnson

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WILLIAM WOOLSEY JOHNSON

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BY

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NEW YORK
JOHN WILEY AND SONS
15 ASTOR PLACE
—
1884

✓ Educ. T 182.84.468

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PRINTED BY J. J. LITTLE & CO.,
ROB. 12 TO 26 ASTOR PLACE, NEW YORK.

PREFACE.

THIS work, as at present issued, is designed as a shorter course in the Integral Calculus, to accompany the abridged edition of the treatise on the Differential Calculus, by Professor J. Minot Rice and the writer. It is intended hereafter to publish a volume commensurate with the full edition of the work above mentioned, of which the present shall form a part, but which shall contain a fuller treatment of many of the subjects here treated, including Definite Integrals, and the Mechanical Applications of the Calculus, as well as Elliptic Integrals, Differential Equations, and the subjects of Probabilities and Averages. The conception of Rates has been employed as the foundation of the definitions, and of the whole subject of the integration of known functions. The connection between integration, as thus defined, and the process of summation, is established in Section VII. Both of these views of an integral—namely, as a quantity generated at a given rate, and as the limit of a sum—have been freely used in expressing geometrical and physical quantities in the integral form.

The treatises of Bertrand, Frenet, Gregory, Todhunter, and Williamson, have been freely consulted. My thanks are due to Professor Rice for very many valuable suggestions in the course of the work, and for performing much the larger share of the work of revising the proof-sheets.

W. W. J.

U. S. NAVAL ACADEMY, *July*, 1881.

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