

**SOLID GEOMETRY;  
PP. 299-481**

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Solid geometry; pp. 299-481 by Various

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**VARIOUS**

**SOLID GEOMETRY;  
PP. 299-481**



# SOLID GEOMETRY

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## PREFACE

IN addition to the features of the Plane Geometry, which are emphasized in the Solid as well, the chief characteristic of this book is the establishment, at every point, of *the vital relation between the Solid and the Plane Geometry*. Many theorems in Solid Geometry have been proved, and many problems have been solved, by reducing them to a plane, and simply applying the corresponding principle of Plane Geometry. Again, many proofs of Plane Geometry have been made to serve as proofs of corresponding theorems in Solid Geometry by merely making the proper changes in terms used. (See §§ 703, 786, 794, 813, 853, 924, 951, 955, 961, etc.)

Other special features of the book may be summarized as follows:

*The student is given every possible aid in forming his early space concepts.* In the early work in Solid Geometry, the average student experiences difficulty in fully comprehending space relations; that is, in *seeing* geometric figures in space. The student is aided in overcoming this difficulty by the introduction of many easy and practical questions and exercises, as well as by being encouraged to *make his figures*. (See § 605.) As a further aid in this direction, reproductions of models made by students themselves are shown in a group (p. 302), and at various points throughout Book VI.

*The student's fund of knowledge is constantly drawn upon.* In the many questions, suggestions, and exercises, his knowledge of the things about him has been constantly appealed to. Especially is this true of the work on the sphere, where the

student's knowledge of mathematical geography has been appealed to in making clear the terms and the relations of figures connected with the sphere.

*The treatment of the Solid Geometry is logical.* The same logical rigor that characterizes the demonstrations in the Plane Geometry is used consistently throughout the Solid. If a postulate is needed to make a proof complete, it is clearly stated, as in § 615. In the mensuration of the prism and the pyramid, the same general plan has been followed as that used in Book IV; in the mensuration of the cylinder, the cone, and the sphere, the method pursued is similar to that used in the mensuration of the circle.

*More proofs and parts of proofs are left to the student* in the Solid, than in the Plane Geometry; but in every case in which the proof is not complete, the incompleteness is specifically stated.

*The treatment of the polyhedral angle* (p. 336), *of the prism* (p. 345), *and of the pyramid* (p. 350), *is similar to that of the cylinder and the cone.* This is in accordance with the recommendations of the leading Mathematical Associations throughout the country.

*The complete collection of formulas of Solid Geometry at the end of the book,* it is hoped, will be found helpful to teacher and student alike.

The grateful acknowledgment of the authors is due to many friends for helpful suggestions; especially to Miss Grace A. Bruce, of the Wadleigh High School, New York; to Mr. Edward B. Parsons, of the Boys' High School, Brooklyn; and to Professor McMahon, of Cornell University.



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## SYMBOLS AND ABBREVIATIONS

<p><math>=</math> equals, equal to, is equal to.</p> <p><math>\neq</math> does not equal.</p> <p><math>&gt;</math> greater than, is greater than.</p> <p><math>&lt;</math> less than, is less than.</p> <p><math>\simeq</math> equivalent, equivalent to, is equivalent to.</p> <p><math>\sim</math> similar, similar to, is similar to.</p> <p><math>\propto</math> is measured by.</p> <p><math>\perp</math> perpendicular, perpendicular to, is perpendicular to.</p> <p><math>\perp\!\!\!\perp</math> perpendiculars.</p> <p><math>\parallel</math> parallel, parallel to, is parallel to.</p> <p><math>\parallel\!\!\!\parallel</math> parallels.</p> <p><math>\dots</math> and so on (sign of continuation).</p> <p><math>\because</math> since.</p> <p><math>\therefore</math> therefore.</p> <p><math>\frown</math> arc; <math>\widehat{AB}</math>, arc <math>AB</math>.</p> <p><math>\square</math>, <math>\square</math> parallelogram, parallelograms.</p> <p><math>\odot</math>, <math>\odot</math> circle, circles.</p> <p><math>\sphericalangle</math>, <math>\sphericalangle</math> angle, angles.</p> <p><math>\triangle</math>, <math>\triangle</math> triangle, triangles.</p>	<p>rt. right.</p> <p>str. straight.</p> <p>ext. exterior.</p> <p>int. interior.</p> <p>alt. alternate.</p> <p>def. definition.</p> <p>ax. axiom.</p> <p>post. postulate.</p> <p>hyp. hypothesis.</p> <p>prop. proposition.</p> <p>prob. problem.</p> <p>th. theorem.</p> <p>cor. corollary.</p> <p>cons. construction.</p> <p>ex. exercise.</p> <p>fig. figure.</p> <p>iden. identity.</p> <p>comp. complementary.</p> <p>sup. supplementary.</p> <p>adj. adjacent.</p> <p>homol. homologous.</p>
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Q.E.D. Quod erat demonstrandum, *which was to be proved.*

Q.E.F. Quod erat faciendum, *which was to be done.*

The signs  $+$ ,  $-$ ,  $\times$ ,  $\div$  have the same meanings as in algebra.

## REFERENCES TO THE PLANE GEOMETRY

**Note.** The following definitions, theorems, etc., from the Plane Geometry which are referred to in the Solid Geometry are here collected for the convenience of the student.

(The numbers below refer to articles in the Plane Geometry.)

**18. Def.** Two geometric figures are equal if they can be made to coincide.

**26.** Two intersecting straight lines can have only one point in common; *i.e.* two intersecting straight lines determine a point.

**34. Def.** A plane surface (or plane) is a surface of unlimited extent such that whatever two of its points are taken, a straight line joining them will lie wholly in the surface.

### ASSUMPTIONS

**54. 1.** Things equal to the same thing, or to equal things, are equal to each other.

2. If equals are added to equals, the sums are equal.

3. If equals are subtracted from equals, the remainders are equal.

4. If equals are added to unequals, the sums are unequal in the same order.

5. If equals are subtracted from unequals, the remainders are unequal in the same order.

6. If unequals are subtracted from equals, the remainders are unequal in the reverse order.

7. (a) If equals are multiplied by equals, the products are equal; (b) if unequals are multiplied by equals, the products are unequal in the same order.

8. (a) If equals are divided by equals, the quotients are equal; (b) if unequals are divided by equals, the quotients are unequal in the same order.

9. If unequals are added to unequals, the less to the less and the greater to the greater, the sums are unequal in the same order.

10. If three magnitudes of the same kind are so related that the first is greater than the second, and the second greater than the third, then the first is greater than the third.