

**MATHEMATICAL  
MONOGRAPHS, NO. 7.  
PROBABILITY AND  
THEORY OF ERRORS**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649344888

Mathematical monographs, No. 7. Probability and Theory of Errors by Robert S. Woodward

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**ROBERT S. WOODWARD**

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## MATHEMATICAL MONOGRAPHS.

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PUBLISHED BY

JOHN WILEY & SONS, NEW YORK.

CHAPMAN & HALL, Limited, LONDON.

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BY

ROBERT S. WOODWARD,  
PRESIDENT CARNEGIE INSTITUTION OF WASHINGTON.

FOURTH EDITION.

FIRST THOUSAND.

NEW YORK:

JOHN WILEY & SONS.

LONDON: CHAPMAN & HALL, LIMITED.

1906.

## EDITORS' PREFACE.

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THE volume called *Higher Mathematics*, the first edition of which was published in 1896, contained eleven chapters by eleven authors, each chapter being independent of the others, but all supposing the reader to have at least a mathematical training equivalent to that given in classical and engineering colleges. The publication of that volume is now discontinued and the chapters are issued in separate form. In these reissues it will generally be found that the monographs are enlarged by additional articles or appendices which either amplify the former presentation or record recent advances. This plan of publication has been arranged in order to meet the demand of teachers and the convenience of classes, but it is also thought that it may prove advantageous to readers in special lines of mathematical literature.

It is the intention of the publishers and editors to add other monographs to the series from time to time, if the call for the same seems to warrant it. Among the topics which are under consideration are those of elliptic functions, the theory of numbers, the group theory, the calculus of variations, and non-Euclidean geometry; possibly also monographs on branches of astronomy, mechanics, and mathematical physics may be included. It is the hope of the editors that this form of publication may tend to promote mathematical study and research over a wider field than that which the former volume has occupied.

December, 1905.

## AUTHOR'S PREFACE.

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IN republishing this short treatise in book form the author solicits criticism but offers no apology. The type of the book he has sought to imitate is that shown in the "mathematical tracts" of the late Sir George B. Airy. The brevity and the concrete illustrations of these "tracts" have served very effectively in introducing students to a number of the more difficult fields of applied mathematics; and it is hoped that this treatise will serve a similar end.

The theory of probability and the theory of errors now constitute a formidable body of knowledge of great mathematical interest and of great practical importance. Though developed largely through applications to the more precise sciences of astronomy, geodesy, and physics, their range of applicability extends to all of the sciences; and they are plainly destined to play an increasingly important rôle in the development and in the applications of the sciences of the future. Hence their study is not only a commendable element in a liberal education, but some knowledge of them is essential to a correct understanding of daily events.

No special novelty of presentation is claimed for this work; but the reader may find it advantageous to know that a definite plan has been followed. This plan consists in presenting each principle, first, by means of a simple, concrete example; passing, secondly, to a general statement by means of a formula; and, thirdly, illustrating applications of the formula by concrete examples. Great pains have been taken also to secure clear and correct statements of fundamental facts. If these latter are duly understood, the student needs little additional aid; if they are not duly understood, no amount of aid will forward him.

The passage from the elementary concrete to the advanced abstract may appear to be abrupt to the reader in some cases. It is hoped, however, that any large gaps may be easily bridged and that any serious difficulties may be easily overcome by means of the references given to the literature of the subject. In any event the student will find that in this, as in all of the more arduous sciences, his greatest pleasure and his highest discipline will come from bridging such gaps and from surmounting such difficulties.

WASHINGTON, D. C., December, 1905.



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1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in the context of public administration and government operations. The text notes that without reliable records, it becomes difficult to track expenditures, assess performance, and ensure that resources are being used effectively and efficiently.

2. The second part of the document addresses the challenges associated with data collection and analysis. It highlights that gathering accurate and timely data can be a complex task, often requiring significant resources and expertise. The text discusses various methods for data collection, including surveys, interviews, and the use of digital tools, and notes that each method has its own strengths and limitations. Additionally, it stresses the importance of ensuring the quality and integrity of the data collected, as well as the need for clear communication and collaboration between different departments and stakeholders.

3. The third part of the document focuses on the role of technology in improving data management and analysis. It discusses how modern software solutions and digital platforms can streamline data collection, storage, and processing, thereby reducing the risk of errors and increasing the efficiency of the data management process. The text also touches upon the importance of data security and privacy, particularly in the context of handling sensitive information. It suggests that organizations should invest in robust security measures and ensure that their data management practices comply with relevant regulations and standards.

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5. The fifth part of the document concludes by emphasizing the need for a culture of data literacy and continuous learning. It suggests that organizations should invest in training and development programs to ensure that their employees have the skills and knowledge needed to effectively use data. The text also notes that data literacy is not just a technical skill, but also a mindset that encourages curiosity, critical thinking, and a willingness to learn from data. Finally, it reiterates the importance of maintaining accurate records and ensuring the quality and integrity of the data used for decision-making.

## PROBABILITY AND THEORY OF ERRORS.

### ART. I. INTRODUCTION.

IT is a curious circumstance that a science so profoundly mathematical as the theory of probability should have originated in the games of chance which occupy the thoughtless and the profligate.\* That such is the case is sufficiently attested by the fact that much of the terminology of the science and many of its familiar illustrations are drawn directly from the vocabulary and the paraphernalia of the gambler and the trickster. It is somewhat surprising, also, considering the antiquity of games of chance, that formal reasoning on the simpler questions in probability did not begin before the time of Pascal and Fermat. Pascal was led to consider the subject during the year 1654 through a problem proposed to him by the Chevalier de Méré, a reputed gambler.† The problem in question is known as the problem of points and may be stated as follows: two players need each a given number of points to win at a certain stage of their game; if they stop at this stage, how should the stakes be divided? Pascal corresponded with his friend Fermat on this question; and it appears that the letters which passed between them contained the earliest distinct formulation of principles falling within the theory of probability. These

\* The historical facts referred to in this article are drawn mostly from Todhunter's *History of the Mathematical Theory of Probability* from the time of Pascal to that of Laplace (Cambridge and London, 1865).

† "Un problème relatif aux jeux de hasard, proposé à un austère janséniste par un homme du monde, a été l'origine du calcul des probabilités." Poisson, *Recherches sur la Probabilité des Jugements* (Paris, 1837).