THE ELEMENTS OF SOLID GEOMETRY. WITH NUMEROUS EXERCISES

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The Elements of Solid Geometry. With Numerous Exercises by Arthur Latham Baker

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WITH NUMEROUS EXERCISES.

BY

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"Every study of a generalization or extension gives additional power over the particular form by which the generalization is suggested."

DE MORGAN, FORMAL LOGIC.

BOSTON, U.S.A.
GINN & COMPANY, PUBLISHERS.
1894.

SUGGESTIONS TO STUDENTS.

When studying a proposition, *draw the figure for yourself on a separate piece of paper*, varying the shape and proportions, but keeping the lettering the same.

Write out the principal points, equations, etc., as you study them.

Consult other books and see if you can find a demonstration that suits you better than the one given here.

When through, write out the demonstration in your own language, using the one which seems simplest and best.

Use pencil and paper constantly. Mathematics must be written into the mind, not read into it. "No head for mathematics" nearly always means "will not use a pencil."

NOTATION.

The following notation is used generally throughout the book, though departed from in some instances where additional brevity could be secured.

Points are indicated by capital letters, A, B, C, M, N, etc., also by the two lines whose intersection determine the point, thus point *pq* means the point at the intersection of the lines *p* and *q*.

Lines are indicated by lower case letters, *a*, *b*, *c*, *m*, *p*, etc.

Planes and surfaces are indicated by capital letters, M, N, R, S, etc.

Angles are indicated by the two lines which form the angle, thus $\angle$ *mn* means the angle between the lines *mn*; or by Greek letters, e.g., α, β, γ, etc.

Plane figures are indicated by giving two or more of their sides, as many as are sufficient to completely determine them. Thus $\triangle$ *ab* means the $\triangle$ two of whose sides are *ab*. Figure *ab* means the figure which has *ab* for its sides. If necessary all the sides are named.

Volumes are denoted by heavy faced type, e.g., **V**, **P**, **Q**, etc.

The ordinary notation of geometry is used where that happens to be shorter and clearer.

The *given parts* are generally designated by the middle letters of the alphabet, the *unknown parts* by the last letters, and the *construction parts* by the first letters of the alphabet in the order in which they are found. Hence a glance at the diagram indicates the given, required, and intermediary parts, and moreover the *order* in which the intermediary parts are found.

Suggestive letters are used where practicable, thus *t* for the intersections (traces) of planes, *h* for height or altitude, *r* for radius, *B* for base.

Corresponding lines in two figures are indicated by corresponding English and Greek letters.

Subscripts indicate a close relation between the parts, equality, similarity of position or homology, etc.

CHAPTER I.

LINES AND PLANES IN SPACE.

Solid Geometry is that branch of geometry in which the forms (or figures) treated are not limited to a single plane.

1. **A plane** is a surface such that a straight line joining any two points in it lies wholly in the surface. A plane is indefinite in extent, so that however far the straight line is produced, all its points lie in the plane; but a limited portion of a plane is usually represented by a parallelogram.

2. A plane is said to be **determined** by certain lines or points, when it is the only plane which contains those lines or points.

3. Any number of planes may be passed through a straight line, for a plane passing through the line may be revolved about the line and made to occupy an infinite number of positions, each of which will be a different plane. Hence a single straight line does not determine a plane.

4. *A plane is determined by three points not in the same straight line.*

For if the plane be turned about the straight line containing two of the points until it contains the third point, the plane is evidently determined, since if it is then revolved either backward or forward, it will no longer contain the third point.

5. *A plane is determined by a straight line and a point without that line, by two intersecting straight lines, or by two parallel lines,* since each of these cases can be converted into that of § 4 by selecting three points, two in one of the given lines and the third in the other line.

6. A straight line is **perpendicular to a plane** when it is perpendicular to every straight line of the plane which passes through its *foot*, that is, the point where it meets the plane.

Conversely, the plane is perpendicular to the line.

7. A line is **oblique** to a plane if it is not perpendicular to all the straight lines drawn in the plane through its foot.

8. A line is **parallel to a plane** when it is the limiting case of an oblique line, that is when its point of intersection has passed out to infinity.

In this case it is said to meet the plane at infinity.

A plane cuts all lines in space which are not parallel to it.

9. The **distance** from a point to a plane is the perpendicular distance from the point to the plane.

10. Two planes are **parallel** when their line of intersection has passed out to infinity; and the planes are said to meet at infinity.

11. The **projection of a point** on a plane is the foot of the perpendicular let fall from the point to the plane.

12. The **projection of a line** on a plane is the locus of the projections of all its points.

13. The **angle which a line makes with a plane** is the angle which it makes with its projection on the plane.

Proposition I. Theorem.

14. *The intersection of two planes is a straight line.*

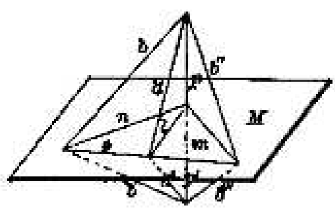
Proof. Join two points of the intersection by a straight line. By definition, § 1, this straight line lies wholly in each plane, and hence must be common to them, or, in other words, be their intersection. Q. E. D.

No point outside of this line can lie in the intersection, for only *one* plane can contain a straight line and a point without, § 5.

Proposition II. Theorem.

15. *If a straight line is perpendicular to each of two straight lines at their point of intersection it is perpendicular to the plane of those lines.*

Notation. Let m and n be two straight lines which determine the plane M, and p another straight line perpendicular to each of them at their intersection. Let l be *any* other straight line in the plane M passing through the intersection of m and n.



To prove $p \perp$ to l, and therefore to the plane M.

Construction. Lay off $m = n$, and prolong p making $p' = p$. Connect the extremities of m, n, p, p' by straight lines. From the intersection of l and one of these new lines, draw lines to the extremities of p and p'.

Analysis. 1°. b, b', b'', b''' are = since points in the $\perp$ bisectors m, n are equally distant from the ends of p, p', and $m = n$. Hence in the isosceles $\triangle$'s $bb''e$, $b'b'''e$, the $\angle\, be = \angle\, b'e$, and the $\triangle\, bed = \triangle\, b'ed'$.

2°. d and d' are $=$ because the $\triangle\, bed = \triangle\, b'ed'$ (having two sides and the included angles equal).

3°. p is $\perp$ to l because the lines d, d' are equal (points equidistant from the ends of a straight line lie in the $\perp$ bisector).

Hence p is perpendicular to M because it is perpendicular to l, any line in that plane. Q. E. D.

Is it necessary that $m = n$?

16. COR. 1. *At a given point in a plane, only one perpendicular to the plane can be erected.*

Otherwise, if we pass a plane through the two perpendiculars, giving an intersection l with the plane M, we should have two $\perp$'s in the same plane to the same straight line l at the same point, which is impossible.

17. COR. 2. *From a point without a plane only one perpendicular can be drawn to the plane.*

For if p, b be two such $\perp$'s, the $\triangle\, pbn$ would contain two rt. $\angle s$, which is impossible.

18. COR. 3. *Oblique lines drawn from a point to a plane and meeting the plane at equal distances from the foot of the perpendicular, are equal.* E.g., b and b'', § 15.

19. COR. 4. *Of oblique lines drawn from a point to a plane the one which meets the plane further from the foot of the perpendicular is the longer.*

If n be extended, b, the hypothenuse of the rt. $\triangle\, bn$, must become longer than b''.

20. COR. 5. *Equal oblique lines from a point to a plane meet the plane at equal distances from the foot of the perpendicular; and of two unequal oblique lines the greater*