ALTERNATE CIRCLES AND THEIR CONNECTION WITH THE ELLIPSE

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EDWARD ADOLPHUS SEYMOUR

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ALTERNATE CIRCLES

AND

THEIR CONNEXION WITH

THE ELLIPSE.

BY THE

DUKE OF SOMERSET.

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1851.

ADVERTISEMENT.

The present Volume is a sequel to the Treatise on the Ellipse.

It consists of Two Sections.

The First Section contains some Geometrical Propositions, demonstrated by an Algebraical Process.

The Second Section employs Alternate Circles to simplify, or to reduce to lower dimensions, such equations as express the properties of the Ellipse.

ALTERNATE CIRCLES.

Section the First.

PROPOSITION I.

PROBLEM.—*To describe two alternate circles, of which the areas shall be in a given ratio, and their sum equal to a given rectangle.*

LET the given ratio be as $a\,a$ to $b\,b$, and the given rectangle $h\,l$.

Let c be the circumference of a circle of which the diameter is unity.

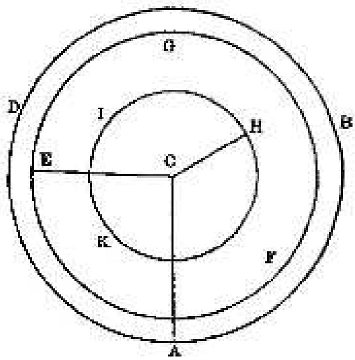
Now, if x be the semi-diameter of a circle, its diameter will be $2\,x$.

Then, $1 : 2x :: c : 2cx$.

Therefore, $2cx$ is the circumference of a circle of which the radius is x.

Then half that circumference is $c\,x$.

Now, because the area of a circle is equal to the rectangle contained by the semi-diameter and a straight line equal to half the circumference, cxx will represent a space equal to the area of a circle of which the semi-diameter is x.



Then, let $kl = cxx$ be equal to the area of the external circle.

Let the semi-diameters of the alternate circles be y and z.

Then, because circles are to one another as the squares of their semi-diameters,

$$yy + zz = xx = \frac{kl}{c},$$

And $yy : zz :: aa : bb$.

$$yy = \frac{aazz}{bb}$$

$$yy = \frac{kl}{c} - zz$$

$$\frac{aazz}{bb} = \frac{kl}{c} - zz$$

$$zz = \frac{bbkl}{(aa + bb)c}$$

$$z = \frac{b\sqrt{kl}}{\sqrt{(aa + bb)c}}$$

$$y = \frac{a\sqrt{kl}}{\sqrt{(aa + bb)c}}$$

$$x = \frac{\sqrt{kl}}{\sqrt{c}}.$$

Then, from any point C, at the distance CA, equal to $\frac{\sqrt{kl}}{\sqrt{c}}$, describe the circle ABD.

From the same point, at the distance CE, equal to $\frac{a\sqrt{kl}}{\sqrt{(aa + bb)c}}$, describe the circle EFG.

From the same point, at the distance CH, equal to $\frac{b\sqrt{kl}}{\sqrt{(aa + bb)c}}$, describe the circle HIK.

The two last circles shall be those required.

For, because $\frac{aakl}{(aa + bb)c} : \frac{bbkl}{(aa + bb)c} :: aa : bb$, the areas of the alternate circles are, one to the other, in the given ratio.

And because $\frac{aakl}{ba + bb} + \frac{bbkl}{aa + bb} = \frac{(aa + bb)kl}{aa + bb} = kl$, the sum of the areas is equal to the given rectangle.

PROPOSITION II.

THEOREM.—*If three straight lines be harmonical proportionals, the area of one alternate circle be equal to a rectangle contained by the first and second, and the area of the other to a rectangle contained by the second and third; the area of the external circle shall be equal to twice the rectangle contained by the first and the third.*

Let the lines LM, MN, and NO, be harmonical proportionals, let the area of the alternate circle EFG be equal to a rectangle contained by LM and MN, and let the area of the alternate circle HIK be equal to a