# ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

#### ISBN 9780649087877

Elements of plane and spherical trigonometry by David A. Rothrock

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd. Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

## **DAVID A. ROTHROCK**

# ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY



# ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY

#### BY

#### DAVID A. ROTHROCK, Ph.D.

PROFESSOR OF MATHEMATICS, INDIANA UNIVERSITY
BLOOMINGTON, INDIANA



THE MACMILLAN COMPANY
1911

All rights reserved

Engineering & Mathematical Sciences Library

#### PREFACE

In this work the author has endeavored to prepare a text which would serve as a basis for a fifty- or sixty-hour course in Plane and Spherical Trigonometry as ordinarily presented in advanced secondary and elementary college courses.

Emphasis is placed upon drill work in the trigonometric identities, upon the applications of trigonometry to practical problems, and upon approximate calculations by means of natural functions. The more accurate results obtained by logarithmic calculations are emphasized in the solutions of oblique triangles; a uniform style of tabulating logarithmic calculations is suggested.

For the benefit of those who may wish to pursue advanced courses in mathematics, a brief discussion of analytic trigonometry is presented in Chapter IX. In Part II the elements of spherical trigonometry are developed in so far as to include the ordinary formulæ necessary in the solution of right and oblique spherical triangles.

DAVID A. ROTHROCK.

Bloomington, Indiana, December, 1909.

¥

## CONTENTS

### PART I

#### PLANE TRIGONOMETRY

#### CHAPTER I

	Trigono	metri	c ru	nctio	113 01	Acu	te Al	rgies	C			
ART.								-				PAGE
1.	Trigonometry .	145	355	4.5	150		2					
2.	Functions of Acute Ar	ngles	.0	. X	30	5			- 6	*		2
3,	Functions of Acute Ar Functions of Complem	entar	y Ai	ngles		(8)			50		8	400
139	Exercises .	14			233	8.0	: E			7.0		
	Fundamental Relation							netic	ns	40	360	- (
	Fundamental Identitie		. Đ.,			18	17	-	65	2.0	*	- 0
0.	Variation of the Trigo	nome	trie I	tunet	ions	*					*	
7.	Transformation of Ide							14			-	1
	Exercises .					+				*	16	1
	Exercises .	55	53	4	38	(*)	+		53	*2		10
8.	Functions of Particula											13
	Exercises .											14
9.	Table of Trigonometri	e Fun	ction	is	(6)	.00	St	100	5	*	*	10
			CIL	APTI	er i	T						
	12											
	S	olutio	on of	Rig	ht T	'riang!	les					
10.	Fundamental Formula		4.1	200	50			4		4		16
11.	Projections	4		60		8		-		*:	*	17
12.	Components	10	00	*	20	37		-	0.00	100	*	18
-0:00 N	Resultant	92	5:3	20	*:	*			1100	20	di:	18
14.	Projected Areas .			20		7		14	100		*	21
15.	Areas of Right Triang			*60				16	0.00	40	10	21
	The Isosceles Triangle		(*)	±13	*	*	25	19		53	*	22
17.	Notations of Direction			83		,				20	20	25
18.	To solve Right Triangl		N.	*				4		40	*	25
	Exercises .	33		90	*	*	95	98	53	10	11	26
			СНА	PTE	R II	II						
	Trigor	omet	ric I	Funct	ions	of An	y A	agle				
19,	Axes. Quadrants			-			,		,		+1	31
20.	Coördinates, Abscissa,	Ordi	nate	10	±7	*					*	31
21.	Definitions of the Fun-	ctions		77		\$	4	7	4	10	+	32

#### CONTENTS

ART.													PAGE
22.	Laws of Signs	0		7	4							- 1	33
1000a	Exercises				26			*					34
23.	Functions of Neg	ative	Ang	les		- 25						4	34
436	Exercises Functions of Neg Exercises Functions of 90° Functions of 180° Exercises Line Values of the	*/	200				•						86
24.	Functions of 90°	- θa	nd 90	00 + 6	9 .								
25,	Functions of 180	$-\theta$	and	180° -	+ θ	4	(90)	*	*		*	3	37
312	Exercises	300	800										38
26,	Line Values of th	e Fu	netic	ns	*			*			0.4		39
	F.xercises	<b>*</b> (0)	903	100		2.30		*				33	41
27,	Graphs of the Tr	gono	metr	ic Fu	netic	ons		*	•	*	ं*		45
				СН	APT	ER J	IV						
			Me	asure	men	t of .	Angles						
28.	Units of Measure	8	•	¥		14							44
29.	Relations between Exercises	n De	gree a	and 1	<b>tadis</b>	m Mi	авите					-	44
	Exercises	95	*	(4)	33	0.9	- 10	40			1.4	3	46
30,	The Length of an	y Ar	c.	(5)	13	07.	1201	*0		*		24	46
31.	The Length of an Segment and Sec	or A	reas	1	-	114				4		75	47
	Exercises	ĕ(	93	*	36	36	(4)	*	*		*	13	47
				CH	APT	ER	v						
	Func	tions	of '	Two	Ang	les.	Multi	ple .	Angli	es			
00	To develop sin (								200				50
99	To develop single	T P)		COM	N. T.	P)	(*)	**		*	٠		51
9.4	To develop tan (6 Important Form	day.	*	蒙		8		ŧ.	8	:		1	52
04.	Exercises		20	43	Ÿ							0.4	52
	Functi			909.930				835	10000	583 SEC.			
35.	Functions of $2\alpha$	1	*		4	-	39		47		19		59
76.	Functions of 3 $\alpha$	E	20	+		· (4)	(0)		*		- 0	135	54
S7.	Half-angle Form		50	*:	*			5	:	*			54
	Exercises	1					14		+			4	54
		s	um a	nd D	iffer	ence	Forme	ilas					
1000	Converting to Pro	ed acces											70
90.	Converting to Su	RIUCE	Thirt.	1	. (7)		•	2					
48974	Exercises											1	57
	Ligercises	*	20	*	*	×.	*		*27	*		((#	01
				CH.	\PT	ER 7	71						
				Lo	gari	thms							
40.	The Index Laws	170	43	2	12	14					112	12	66
41.	The Index Laws Definition of Log	arith	ms	(4)		14	-				14	24	61

			CO	NTE	NTS	i						îx
ART,												PAGE
42.	Systems of Logarithm.	š.	200	314	(6)	***				39.1	0.00	62
43.	Laws governing the U	se of	Log	arithn	18							200
44.	Characteristic and Ma	ntissa										63
45.	Use of Tables ,	(9)	0.00	338	60	80	**	946				65
46.	Conversion of Commo	n to 2	Vapi	erian	Logar	rithn	as.			3		67
	Use of Tables Conversion of Commo Exercises in Use of	of Log	carit	hms	•11	2	*		•	4	•	67
		a - 3	CIL	APTE	R V	11						
	Sol	ution	of	Trian	gles .	in G	enera	1				
47.	The Theorem of Sines				Tirraci Ta	160 TE		801 180				69
	Applications of th	e The	OTH	m of S	Sines	200	.00			55		
48.	The Theorem of Tange	ents				- 0	촳	·				
	Applications of th	e The	ore	m of T	Cance	mts	3.00	38				74
49.	The Theorem of Cosin		::+							2.5		121
231	Exercises .		Œ.	77.0				摄	- 3			100
50.	The Half-angle Theore	eme		70.00					84			
	Application of the						,	*	3.5	33		71
		A	reas	of T	'riane	les	8					
51.	Area in Terms of Side				20040616006	9.000	2	333			251	80
	Area in Terms of r				¥8				-			81
52.	Expressions for Area			1000							#11	8:
7555	아이면 어려움을 통하면 있다면 없는 아이들이 하다니다. 살으로				- 13		:	•	ij.			82
		,	°H/	APTE	R V	111						
	Inverse F						c Ear	uatio	ns			
64	**************************************			12-3005	54,100	1980						87
				500		200			1		- 3	88
00.	Exercises .	35	2			3		*				81
. 0						80						
			3.			*					23	90
01.	Solutions Simple Equations .	1	3		•					12.1	12	97)
98.	Simple Equations .		1									
	Exercises . Equations of the Form	o Maria			100 m	1				(0)	0.0	95
00.	Equations of the Form	1 1, 4500	8 p =	= a, r	sin φ	= 0					1	90
60.	Equations of the Form	1.3, 810	W Co	$18 \varphi =$	a, re	an d	san ø	= 0, 1	r cos	2 = c		299
61,	To solve $a \cdot \sin x + b \cdot c$	008 X =	- C	66	+1.	+			19	69	0	1/9
62,	To solve $\sin(x+\phi) =$	a sin	5		900	*	30					98
	To solve $\tan (x + \phi) =$				3							94
64.	To solve $x = \alpha + \beta \sin x$	x.		6	1	4	*	1	74	-	4	9
			СН	APTE	R I	X						
1	Complex Numbers.	DeM	oiv	re's T	heore	m.	Trig	onom	etric	Serie	es.	
	Expon	entia	l an	d Hy	perbo	lic E	unct					
	Roots of Quadratic Eq											
	Exercises .		114	13.5		1			374	100	100	100

		18.7		N.	THE S
	100	11	150	100	

x		CO	NIE	NIS							
ART.		on zane ez.									PAGE
66.	Complex Numbers express	ed Tri	gono	metri	cally		*:	*		*	100
67.	DeMoivre's Theorem . Raising to Powers and ext		1.10		114			*		14	101
68,	Raising to Powers and ext	ractin	g Ro	ota				*	*	1.0	102
69.	Value of sin x, cos x in Ter Summation of Series . Examples	ms of	x		33 1			*	18	65	104
70.	Summation of Series .	70	*	*						3	108
	Examples	*	*		74						105
71.	The Exponential Series.	3.5	*				*:	*	*		110
72.	Euler's Formulas	18	*				*			1.0	111
22	Exercises			-			*5			*	112
76.	The Hyperbolic Functions	* *	*		19		*	*	*	. 4	112
74.	Examples . The Exponential Series . Enter's Formulas . Exercises . The Hyperbolic Functions The Gudermannian .	80	*	*		•	***	*		*	116
		$\mathbf{P}^{A}$	RT	11							
	SPHERIC	CAT	TR	IGO	NON	ET	RV				
	52 22570										
	General Def			ER N		T-:	nnela				
					202000		00.000				(5),598
	Definitions and Geometric					3	• 63	*		3.	117
76.	The Polar Triangle .	¥6	*	*	2	d	4	*	*	*	118
	The R	·				-					
77.	Definitions	200	4		\$			4		3.0	119
78.	Trigonometric Relations	90	96	0.0		9	*10			*	119
79.	Important Formulas .	*	*		25		9.5		(4)	18	120
80.	Napler's Rules of Circular	Parts			4				100	100	121
81.	Relative Dimensions of Sic	les an	d An	gles					*	10	122
82.	The Isosceles and Quadrat	ital Ti	riang	les	98	•		*1		1.5	122
83.	Solution of Right Spherica	1 Tria:	ngles						7		
	Trigonometric Relations important Formulas Napier's Rules of Circular Relative Dimensions of Sir The Isosceles and Quadrar Solution of Right Spherica Exercises			34	4		•11	*		34	124
		CHA	PTE	R X	I						
	The Ob	lique	Sph	erical	Tria	ngle	1				
84.	The Theorem of Sines .		¥	2	2					10	126
85.	The Theorem of Cosines.  Modified Formula .	Side	and (	Опе А	100						127
	Modified Formula .	**	*	-90				90		4	128
86.	The Theorem of Cosines.	Angle	es an	d One	Side			,			128
87.	The Theorem of Cosines.  Modified Formula  The Theorem of Cosines.  The Half-angle Formulae  1) The Half-angles in T  2) The Half-sides in Te  Napier's Analogies  The Area of a Spherical T  Solution of Oblique Spheri  Exercises				2.4					2.0	125
	1) The Half-angles in T	erms (	of the	Side	s.	(*)	• 1	*:			129
	<ol><li>The Half-sides in Te</li></ol>	rms of	the	Angle	s		• 70				130
88.	Napier's Analogies .	N. Carrier			12					1	131
89.	The Area of a Spherical T	riangle	е.		174						133
90.	Solution of Oblique Spheri	cal Tr	iangl	es	106	(0)			*		133
	Exercises , ,										135

	CONTENTS											2		
			Appli	catio	ons of	Sphe	rica	Trig	опоп	etry				
91.		Earth as	a Sphe	re.	Defini	tions	and	Notat	ion	32		5	137	
92.	The T	Cerrestri:	al Tria	ngle				7					138	
93.	The (	Celestial	Sphere		135	*3	80	(8)		1996	(100)	90	140	
94.	The (	Celestial	Triang	le .							+	7	141	
	E	ercises				+1	\$		4	-			142	
					СН	APTI	ER :	ш						
For	MULAS	6 8 3	36	:22	88	8	į.		1,3	8			143	