ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649087877

Elements of plane and spherical trigonometry by David A. Rothrock

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DAVID A. ROTHROCK

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PREFACE

In this work the author has endeavored to prepare a text which would serve as a basis for a fifty- or sixty-hour course in Plane and Spherical Trigonometry as ordinarily presented in advanced secondary and elementary college courses.

Emphasis is placed upon drill work in the trigonometric identities, upon the applications of trigonometry to practical problems, and upon approximate calculations by means of natural functions. The more accurate results obtained by logarithmic calculations are emphasized in the solutions of oblique triangles; a uniform style of tabulating logarithmic calculations is suggested.

For the benefit of those who may wish to pursue advanced courses in mathematics, a brief discussion of analytic trigonometry is presented in Chapter IX. In Part II the elements of spherical trigonometry are developed in so far as to include the ordinary formulæ necessary in the solution of right and oblique spherical triangles.

DAVID A. ROTHROCK.

BLOOMINGTON, INDIANA,
December, 1909.

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