

**A TREATISE ON THE  
PRINCIPLES AND  
APPLICATIONS OF  
ANALYTIC GEOMETRY**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649154876

A treatise on the principles and applications of analytic geometry by Henry T. Eddy

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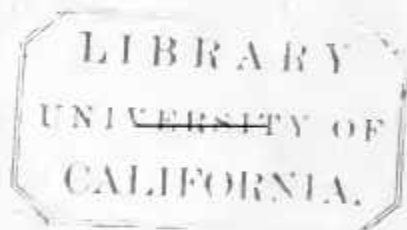
**HENRY T. EDDY**

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OF  
ANALYTIC GEOMETRY.

BY  
HENRY T. EDDY, C.E., PH.D.,  
PROFESSOR OF MATHEMATICS AND ASTRONOMY IN THE UNIVERSITY  
OF CINCINNATI.



PHILADELPHIA:  
COWPERTHWAIT & COMPANY.  
1874.

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*Entered according to Act of Congress, in the year 1874, by*

*HENRY T. EDDY,*

*In the Office of the Librarian of Congress, at Washington.*

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WHYCOFF & THOMSON,  
*Stereotypers and Electrotypers, Phila.*

EDMUND DEACON,  
*Printer, Phila.*

## P R E F A C E.

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THE following treatise, designed as a text-book upon Analytic Geometry, has been written with the most practical ends in view, and is intended to meet the wants of classes in Scientific and Technological Schools, Colleges and Universities. While the needs of the student of Mechanics, Astronomy and Civil Engineering have never been forgotten, it has been found possible to so select the material and to put it in such shape as to adapt the work to the student who pursues the subject merely as a part of a liberal education.

The prime difficulty the ordinary student meets in the study of analytic geometry is in the use of *variables*, since with these he has had no previous acquaintance.

No pains has been spared to make the introduction to their use clear and free from all other complexities. To this end a thorough knowledge of co-ordinates has been first secured by the study of the relations of points, the transformation of co-ordinates, etc.

Again, the entire subject of the general relation of constant and variable quantities is postponed to Chapter V, at which point the student will have attained a sufficient acquaintance with the processes and notation peculiar to analytic geometry to grasp the ideas advanced and use them in after work.

To secure an accurate knowledge of the meaning of the general equations, it is essential that the student should solve numerous numerical examples. They should be illustrations, and of such simple character as to be readily solved by any one who understands the preceding text.

Such are the examples interspersed through the work, which should in no case be omitted. Indeed, if the class is numerous, the teacher is advised to largely increase the number of examples as class-room work by substituting other numbers than those used, and giving each example to a sufficient number of different computers to ensure correct results.

The *Exercises* are much more difficult than the examples, and have two objects in view: first, as original work for the more ambitious students; and secondly, as results to be referred to in the students' subsequent studies. They may be omitted by the ordinary student.

The great difficulty which the teacher experiences is not usually that the

student cannot be made to apprehend the true import of the demonstrations, but it is this—that he afterward fails to recall the necessary equations and their significance.

To assist the teacher in this vital point, the statement of each theorem is in a form to be memorized, and contains some important equation and its signification. The importance of acquiring a perfect familiarity with these statements in algebraic language instead of ordinary language cannot be too strongly emphasized. It has been found by the best teachers that ten or fifteen minutes during every recitation hour should be spent in reciting from memory the statements of all theorems previously studied.

The form of notation adopted is thoroughly systematized, and prepares the student to read with ease the great modern writers upon analytic geometry. The marked value of the angular notation used is a sufficient recommendation for its adoption. For it I am happy to acknowledge my indebtedness to Prof. J. M. Peirce, of Harvard University, from whose works it is borrowed.

The one great defect of text-books upon analytic geometry is the omission of general principles. It appears to be assumed that an acquaintance with its ordinary processes gives the student a knowledge of its principles. This is far from being a correct assumption, as an examination of the general principles stated and demonstrated in Chapter V will abundantly show.

The general discussion of curves and their singularities by means of their approximate equations—a method due to the genius of Newton—is here for the first time rendered accessible to the ordinary student, and it is thought that it will serve a most useful purpose by putting into his hands an instrument of research of practical value whose power compares favorably with the resources of the differential calculus.

No attempt has been made in the last chapters to follow the beaten track of previous text-books, but rather to select matter respecting spirals, etc., of the greatest use to the student.

The book will be found to be suited to the wants of classes taking either a longer or shorter course by the various omissions indicated in the course of it.

I take this opportunity to express my thanks to Prof. James Edward Oliver, of Cornell University, for many happy suggestions.

I am especially indebted to Prof. E. W. Hyde, formerly of Cornell University, who has with signal ability and fidelity assisted me in preparing the book for the press. In particular, the *Examples* were, almost without exception, made by him.

Part Second, upon Solid Geometry, is in preparation, and will be issued at as early a date as circumstances may permit.

—HENRY T. EDDY.



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## *Abbreviations and Signs.*

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**E. G.** is used to introduce an illustrative example.

**N. B.** is used to introduce some useful notation or convention adopted.

A *period* may signify *multiplied by*, and a *colon* may signify *divided by*.

$\pi = 3.14159$  is the semi-circumference of the circle whose radius is unity.

$\therefore$  signifies *therefore*, and *i. e.* signifies *that is*.

$OPQ$  may signify *the angle OPQ*, etc.

The dagger (†) signifies that the proposition to which it is affixed may be omitted if desirable.

$\phi(x, y)$  signifies some unknown function of  $x$  and  $y$ , and is read *phi function of  $x$  and  $y$* .

Read combinations of subscripts, primes and powers as follows:  $P_1$ , *pe one*;  $P_2$ , *pe two*;  $P'$ , *pe prime*;  $P''$ , *pe second*;  $P_1''$ , *pe one second*;  $P'_2$ , *pe prime two*;  $x'^2$ , *ex prime square*;  $x_2^2$ , *ex two square*, etc.

$\frac{y}{x}$  signifies *the angle between  $x$  and  $y$*  (Art. 12).

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## *Greek Alphabet.*

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$\alpha$ alpha.	$\iota$ iota.	$\rho$ rho.
$\beta$ beta.	$\kappa$ kappa.	$\sigma$ c sigma.
$\gamma$ gamma.	$\lambda$ lambda.	$\tau$ tau.
$\delta$ delta.	$\mu$ mu.	$\upsilon$ upsilon.
$\epsilon$ epsilon.	$\nu$ nu.	$\phi$ phi.
$\zeta$ zeta.	$\xi$ xi.	$\chi$ chi.
$\eta$ eta.	$\omicron$ omicron.	$\psi$ psi.
$\theta$ theta.	$\pi$ pi.	$\omega$ omega.