THE AXIOMS OF DESCRIPTIVE GEOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649346875

The Axioms of Descriptive Geometry by A. N. Whitehead

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by

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CAMBRIDGE:
at the University Press
1907

Cambridge:
PRINTED BY JOHN CLAY, M.A.
AT THE UNIVERSITY PRESS.

PREFACE.

THIS tract is written in connection with the previous tract, No. 4 of this series, on Projective Geometry, and with the same general aims. In that tract, after the statement of the axioms, the ideas considered were those concerning harmonic ranges, projectivity, order, the introduction of coordinates, and cross-ratio. In the present tract, after the statement of the axioms, the ideas considered are those concerning the association of Projective and Descriptive Geometry by means of ideal points, point to point correspondence, congruence, distance, and metrical geometry. It has been my object in both tracts to extend the investigations just far enough to assure the reader that the whole of Geometry is really secured by the axioms stated. My hopes for a comparative freedom from typographical errors are based upon my experience of the excellence of the University Press.

A. N. W.

CAMBRIDGE.
March, 1907.

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CHAPTER I.

FORMULATIONS OF THE AXIOMS.

1. The general considerations which must govern a mathematical investigation on the foundations of Geometry have been explained in Chapter I of the previous tract of this series, on the Axioms of Projective Geometry*. It is explained there that 'Descriptive Geometry' is here used as a generic term for any Geometry in which two straight lines in a plane do not necessarily intersect. Also it is pointed out that the purely classificatory portions of a Descriptive Geometry are clumsy and uninteresting, and that accordingly the idea of order is introduced from the very beginning.

There are three main ways by which this introduction of order can be conveniently managed. In one way, which is represented by Peano's axioms given below (§§ 3—6), the *class* of points which lie *between* any two points is taken as a fundamental idea. It is then easy to define the class of points collinear with the two points and lying *beyond* one of them. Thus these three classes of points, namely the two classes lying beyond the two points respectively and the class lying between the two points, together with the two points themselves form the straight line defined by the two points. Then a set of axioms of the straight line are required, concerned with the ides of 'between,' and also axioms are required respecting coplanar lines.

Another way, which was pointed out by Vailati† and Russell‡, is to conceive a straight line as essentially a serial relation involving two terms. The whole field of such a relation, namely the terms which are thus ranged in order by it, forms the class of points on the straight line. Thus the Geometry starts with the fundamental conception of a

* In the sequel this tract will be referred to as 'Proj. Geom.'

† Cf. *Rivista di Matematica*, vol. IV.

‡ Cf. *Principles of Mathematics*, § 376.