AN ELEMENTARY TREATISE ON THE INTEGRAL CALCULUS, FOUNDED ON THE METHOD OF RATES OR FLUXIONS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649570867

An Elementary Treatise on the Integral Calculus, Founded on the Method of Rates or Fluxions
by William Woolsey Johnson

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WILLIAM WOOLSEY JOHNSON

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NEW YORK:
JOHN WILEY AND SONS,
53 EAST TENTH STREET,
1892.

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PREFACE.

THIS work, as at present issued, is designed as a shorter course in the Integral Calculus, to accompany the abridged edition of the treatise on the Differential Calculus, by Professor J. Minot Rice and the writer. It is intended hereafter to publish a volume commensurate with the full edition of the work above mentioned, of which the present shall form a part, but which shall contain a fuller treatment of many of the subjects here treated, including Definite Integrals, and the Mechanical Applications of the Calculus, as well as Elliptic Integrals, Differential Equations, and the subjects of Probabilities and Averages. The conception of Rates has been employed as the foundation of the definitions, and of the whole subject of the integration of known functions. The connection between integration, as thus defined, and the process of summation, is established in Section VII. Both of these views of an integral—namely, as a quantity generated at a given rate, and as the limit of a sum—have been freely used in expressing geometrical and physical quantities in the integral form.

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The treatises of Bertrand, Frenet, Gregory, Todhunter, and Williamson, have been freely consulted. My thanks are due to Professor Rice for very many valuable suggestions in the course of the work, and for performing much the larger share of the work of revising the proof-sheets.

W. W. J.

U. S. NAVAL ACADEMY, *July*, 1881.

CONTENTS.

CHAPTER I.

ELEMENTARY METHODS OF INTEGRATION.

I.

PAGE

Integrals 1
The differential of a curvilinear area 3
Definite and indefinite integrals 4
Elementary theorems 6
Fundamental integrals 7
Examples I 10

II.

Direct integration 14
Rational fractions 15
Denominators of the second degree 16
Denominators of degrees higher than the second 19
Denominators containing equal roots 22
Examples II 26

III.

Trigonometric integrals 33
Cases in which $\int \sin^m \theta \cos^n \theta \, d\theta$ is directly integrable 34
The integrals $\int \sin^2 \theta \, d\theta$, and $\int \cos^2 \theta \, d\theta$ 36
The integrals $\int \frac{d\theta}{\sin \theta \cos \theta}$, $\int \frac{d\theta}{\sin \theta}$, and $\int \frac{d\theta}{\cos \theta}$ 37

PAGE

Miscellaneous trigonometric integrals ... 38

The integration of $\frac{d\theta}{a + b\cos\theta}$ ... 40

Examples III ... 43

CHAPTER II.

METHODS OF INTEGRATION—CONTINUED.

IV.

Integration by change of independent variable ... 50
Transformation of trigonometric forms ... 51
Limits of a transformed integral ... 53
The reciprocal of x employed as the new independent variable ... 53
A power of x employed as the new independent variable ... 54
Examples IV ... 56

V.

Integrals containing radicals ... 59
Radicals of the form $\sqrt{(ax^2 + b)}$ ... 61

The integration of $\frac{dx}{\sqrt{(x^2 \pm a^2)}}$ ... 64

Transformation to trigonometric forms ... 65
Radicals of the form $\sqrt{(ax^2 + bx + c)}$ ... 67

The integrals $\int \frac{dx}{\sqrt{[(x-\alpha)(x-\beta)]}}$ and $\int \frac{dx}{\sqrt{[(x-\alpha)(\beta-x)]}}$ ... 68

Examples V ... 70

VI.

Integration by parts ... 77
A geometrical illustration ... 78
Applications ... 78
Formulas of reduction ... 81

Reduction of $\int \sin^m\theta\, d\theta$ and $\int \cos^m\theta\, d\theta$ ... 82

Reduction of $\int \sin^m\theta \cos^n\theta\, d\theta$ ... 84

PAGE
Illustrative examples 87
Extension of the formula employed in integration by parts 89
Taylor's theorem 90
Examples VI 91

VII.

Definite integrals 97
Multiple-valued integrals 100
Formulas of reduction for definite integrals 101
Elementary theorems relating to definite integrals 104
Change of independent variable in a definite integral 105
The differentiation of an integral 106
Integration under the integral sign 109
The definite integral regarded as the limiting value of a sum 111
Additional formulas of integration 115
Examples VII 117

CHAPTER III.

GEOMETRICAL APPLICATIONS.

VIII.

Areas generated by variable lines having fixed directions 123
Application to the witch 124
Application to the parabola when referred to oblique coordinates 126
The employment of an auxiliary variable 126
Areas generated by rotating variable lines 128
The area of the lemniscata 129
The area of the cissoid 130
A transformation of the polar formulas 130
Application to the folium 131
Examples VIII 134

IX.

The volumes of solids of revolution 141
The volume of an ellipsoid 143
Solids of revolution regarded as generated by cylindrical surfaces 144
Double integration 145
Determination of the volume of a solid by double integration 149