A KEY TO THE INTELLECTUAL ARITHMETIC

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BENJAMIN GREENLEAF

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TO THE

INTELLECTUAL ARITHMETIC,

CONTAINING

ANSWERS AND SOLUTIONS OF THE MORE DIFFICULT
EXAMPLES IN THAT WORK.

FOR THE USE OF TEACHERS.

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PREFACE.

SINCE the publication of the Intellectual Arithmetic, it has been found expedient to prepare a Key, to lighten the labor of teachers. It would, however, be impracticable and unnecessary to furnish a full analysis of every example. The solutions which involve new principles are given in full, unless those furnished in the Arithmetic itself form a sufficient guide; if other examples require any attention, either the answers alone are given, or the leading points of the analysis are briefly expressed, using signs whenever it is practicable, and contracting more than would be allowable in recitation, even by the most advanced pupils.

It will be observed, that, in using the sign $\times$, the multiplier is sometimes placed before, and sometimes after it.

As this work is intended for the use of *teachers* only, language is sometimes used in the discussion of principles which young pupils might not understand, and occasionally more than one mode of analysis is given, leaving the teacher to select the one which will be most consistent with his own method of teaching.

Lessons LII. – LXIII. were prepared for the convenience of those teachers who wish to train their pupils to perform mental operations on large numbers. Others can omit any portion of these lessons if they choose to do so, as such a course would not mar the unity of the work.

MAY 15, 1858.

KEY

TO

GREENLEAF'S

INTELLECTUAL ARITHMETIC.

LESSONS I. - X. treat of Addition and Subtraction, and require no explanation here.

LESSON XI. PAGE 24, NOTE. This formula may be introduced by either *since*, *as*, *if*, or a similar conjunction, or otherwise varied to suit the taste of different teachers. The reasoning, fully expressed, would include the fact that 2 apples cost twice as much as one apple. The pupil should understand that the suppressed proposition is always understood; but it is so intimately connected with the very idea of number, and so readily supplied, that it may not be necessary always to express it, any more than it is always necessary to express both the premises of every syllogism. A formula which is used so constantly, should be as brief as possible, provided it be at the same time accurate.

LESSON XIII. — PAGES 28 – 31.

10. If one travels 3 and the other 4 miles an hour, at the end of one hour they will be $3 + 4 = 7$ miles apart; in 5 hours they will be 5 times 7 miles, which are 35 miles, apart.

19. If one travels 4 and the other 3 miles an hour, in one hour they will be $4 + 3 = 7$ miles nearer each other; in 5 hours they will be 5 times 7 miles, or 35 miles, nearer each other. They were 50 miles apart when they started, hence the distance between them will be the difference between 50 and 35 miles, which is 15 miles.

37. $ 175. [25 × 6 + 25]
38. 96 feet. [(12 — 4) × 12]
45. 20 quarts. [64 — (24 + 20)]
46. 80 chickens. [(3 + 15 + 22) × 2]
47. 30 nuts. [75 — (20 + 25)]
48. Each obtained 52 cents.

LESSON XVIII. — PAGES 42–44.

16. 63 cents are 7 times 9 cents; therefore, for 63 cents there can be bought 7 times 4 oranges. Or it might be stated thus: — As many times 4 oranges can be bought for 63 cents, as 9 cents is contained times in 63 cents; 9 is contained in 63, 7 times; therefore there can be bought 7 times 4 oranges, which are 28 oranges.

17. $ 28. [1 ton costs $ 7, and 4 tons cost 4 times $ 7.]

18. 3 cents. [(60 ÷ 4) × 3 = 45; (60 ÷ 5) × 4 = 48; 48 — 45 = 3.]

26. If it take 4 men 8 days, it will take 1 man 4 times 8 days, which are 32 days; if it take 1 man 32 days, it will take 16 men one sixteenth of 32 days, which is 2 days.

27. If it require 16 men to do it in 2 days, it will require 2 times 16 men, or 32 men, to do it in one day, and it will require one eighth of 32 men to do it in 8 days. Or,

If it take 16 men 2 days, it will take 1 man 16 times 2 days, or 32 days, and if it take 1 man 32 days, it will take as many men to do it in 8 days as 8 days is contained times in 32 days.*   Ans. 4 men.

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* It will be observed that, in dividing 32 by 8, we use different expressions in the two solutions here given. Where the numbers are of different denominations, and the dividend is of the same kind as the result, we may use the expression, "one eighth of 32 men"; but where the denominations are the same, we must say, "as many as 8 days is contained times in 32 days." In the latter case the divisor and dividend are both denominate and the quotient abstract; but the former is really multiplying by a fraction, the multiplicand and product (or dividend and quotient) being denominate, and the multiplier (or divisor) abstract. Several examples in Lessons XIV., XVI.,

29. 6 days. $[(9 \times 8) \div 12]$
31. 15 hours. $[30 \div (7 - 5)]$
32. 15 minutes. $[(40 \div 4) - (40 \div 5) = 2; 30 \div 2 = 15]$
33. $32. $[(12 + 20 + 64) \div 3]$
34. $5. $[(18 \div 3) \times 10 \div 12]$
35. 9 bushels. $[27 \times 3 \div 9]$

36. If 1 bushel of corn is worth 2 bushels of oats, 2 bushels of corn are worth 2 times 2 bushels of oats, which are 4 bushels of oats; hence 4 bushels of oats are worth 1 bushel of wheat, and 20 bushels of oats are worth as many bushels of wheat as 4 is contained times in 20.　　Ans. 5 bushels.

37. 3 days.　　| 38. 9 pounds. $[81 \div (3 \times 3)]$

39. If 10 gallons run in and 5 gallons run out in an hour, 5 gallons remain in the cistern, and it will take as many hours to fill it as 5 gallons is contained times in 60 gallons.

Ans. 12 hours.

LESSON XIX.—PAGES 45, 46.

UNITED STATES MONEY.

1. Since in 1 cent there are 10 mills, in 2 cents there will be 2 times 10 mills, which are 20 mills. Or,

There are 2 times as many mills in 2 cents as in 1 cent; in 1 cent there are 10 mills, and in 2 cents there will be 2 times 10 mills, which are 20 mills. Therefore, in 2 cents there are 20 mills.

All examples in Reduction Descending may be explained in a manner similar to one of the above forms.

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and XVII. might be solved by the former method, as well as the one given in Lesson XIV.

The 26th question might also be solved by finding how many men it would take to do it in one day. All similar examples may be explained in these two ways; but perhaps it is as well to select the method which makes the intermediate results of the same denomination as the answer is to be. The facts that it will take 16 men ¼ as long as it takes 4 men, and will require ¼ as many men to do the work in 8 days as to do it in 2 days, give a third solution of questions 26 and 27.

2. As there is 1 cent in 10 mills, there will be as many cents in 20 mills as 10 mills is contained times in 20 mills, which are 2.   Ans. 2 cents.

All examples in Reduction Ascending may be explained in a similar manner.

In 47 mills there are 4 cents, and 7 mills remaining.

4. In 43 cents there are 4 dimes, and 3 cents remaining.

10. If 1 yard costs 5 mills, 20 yards will cost 20 times 5 mills, which are 100 mills. As there is 1 cent in 10 mills, there will be as many cents in 100 mills as 10 mills is contained times in 100 mills, which are 10.   Ans. 10 cents.

11. $6. [(2 × 30) ÷ 10]

12. 25 lbs. [(10 × 5) ÷ 2]

13. If 7 lbs. cost 70 cents, 1 pound will cost one seventh of 70 cents, which is 10 cents. There are 10 dollars in 1 eagle; in 1 dollar there are 100 cents, and in 10 dollars there will be 10 times 100 cents, which are 1,000 cents. If 1 pound costs 10 cents, we can buy as many pounds for 1,000 cents as 10 cents is contained times in 1,000 cents, which are 100.   Ans. 100 lbs.

ENGLISH MONEY.

8. Since in 1 pound there are 20 shillings, in 2 pounds there will be 2 times 20 shillings, which are 40 shillings, and 6 shillings added give 46 shillings.

9. 6 shillings. [(3 × 24) ÷ 12]

10. 8£. [(5 × 32) ÷ 20]

LESSON XX.—PAGES 46–48.

TROY WEIGHT.

9. $5.40. [6 × (20 × 4 + 10)]

10. 405 dimes. [9 × (20 × 2 + 5)]

AVOIRDUPOIS WEIGHT.

9. $240. [20 × 6 × 2]

10. 168 lbs. $[25 \times 6 + 13]$
11. \$22.50. $[(4 \times 2 + 1) \times 25 \times 10]$
12. \$180. $[20 \times 3 \times (15 \div 5)]$

LESSON XXI. — Pages 48-50.

Long Measure.

12. 96 fur.; 144 fur.
13. 40 m.; 120 m.; 240 m.

14. 102 in.; 65 in.

Cloth Measure.

7. \$80.
8. 61 nails.
9. 23 quarters.

10. 5 yd., and 3 qr. rem.
11. \$1.80.
12. \$4.

LESSON XXII — Pages 50, 51.

Square Measure.

8. 108 sq. rods. | 9. \$800. | 10. 260 sq. rods.

Solid Measure.

8. \$30. $[3 \times 2 \times 5]$
9. \$25. $[(40 \times 2 + 20) \div 4]$

LESSON XXIII. — Pages 52, 53.

Liquid Measure.

8. \$1.60; \$2.56; \$3.20.
9. 24 cts.; 12 cts.: 9 cts.
10. \$1.28.

11. \$3.
12. 20 gills.
13. \$42. $[3 \times (70 \div 5)]$

Dry Measure.

7. 96 cts.
8. \$1.28.
9. 64 qts.; 96 qts.

10. 184 pints.
11. \$3.20.