ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY; WITH PRACTICAL APPLICATIONS

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BENJAMIN GREENLEAF

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WITH

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By BENJAMIN GREENLEAF, A. M.,

AUTHOR OF A MATHEMATICAL SERIES.

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Complete Keys to the Intellectual, Common School, and National Arithmetics, Algebras, Geometry and Trigonometry ; *for Teachers only.*

PREFACE.

THE preparation of this treatise was undertaken at the earnest solicitation of many teachers, who, having used the author's Arithmetics, Algebras, and Elements of Geometry with satisfaction, were desirous of seeing his series rendered more nearly complete by the addition of the Elements of Plane and Spherical Trigonometry. It is hoped that the present work will be found to be a complete system, theoretical and practical, fully adapted to the wants of advanced classes.

The trigonometric functions, in this treatise, have been regarded as *ratios*, since this improved method has not only now superseded the ancient method in English and French works, but has been approved and adopted generally by the best American mathematicians. Reference, however, is made to whatever is especially valuable in the old method.

In the preparation of this work the author has received valuable suggestions from many eminent teachers, to whom he would here express his sincere thanks. Especially would he acknowledge his great obligations to H. B. Maglathlin, A. M., who for many months has been associated with him in his labors, and to whose experience as a teacher, skill as a mathematician, and ability as a writer, the value of this treatise is largely due.

<div align="right">BENJAMIN GREENLEAF.</div>

BRADFORD, Mass., July 25, 1861.

NOTICE.

THIS System of Trigonometry was designed to be published in connection with the author's Elements of Geometry. It is now published *separately*, to accommodate those teachers who desire to use it in that form.

January 1, 1863.

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TRIGONOMETRY.

ELEMENTS

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TRIGONOMETRY.

BOOK I.

LOGARITHMS.

1. THE LOGARITHM of a number is the exponent of the power to which a given fixed number must be raised in order to produce the first number.

2. The BASE of the system is the fixed number.

3. The base, in the common system of logarithms, is 10. Hence, since

$10^0 =$	1,	0	is the logarithm of		1 ;
$10^1 =$	10,	1	"	"	10 ;
$10^2 =$	100,	2	"	"	100 ;
$10^3 =$	1,000,	3	"	"	1,000 ;
$10^4 =$	10,000,	4	"	"	10,000 ;
	&c.,			&c.	

It thus appears that, in the common system, the logarithm of every number between 1 and 10 is some number between 0 and 1 ; that is, a proper fraction. The logarithm of every number between 10 and 100 is some number between 1 and 2 ; that is, 1 plus a fraction. The logarithm of every number between 100 and 1,000 is some number between 2 and 3 ; that is, 2 plus a fraction ; and so on.

4. By means of *negative* exponents the application of logarithms may be extended, in the common system, to numbers less than 1. Thus, since

$$10^{-1} = 0.1, \quad -1 \text{ is the logarithm of } 0.1;$$
$$10^{-2} = 0.01, \quad -2 \text{ " " } 0.01;$$
$$10^{-3} = 0.001, \quad -3 \text{ " " } 0.001;$$
$$10^{-4} = 0.0001, \quad -4 \text{ " " } 0.0001;$$
$$\&c., \quad \&c.$$

From this it appears that the logarithm of every number between 1 and 0.1 is some number between 0 and —1; that is, —1 plus a fraction. The logarithm of every number between 0.1 and 0.01 is some number between —1 and —2; that is, —2 plus a fraction. The logarithm of every number between 0.01 and 0.001 is some number between —2 and —3; that is, —3 plus a fraction; and so on.

5. In the common system, as the logarithms of all numbers which are not exact powers of 10 are incommensurable with those numbers, their values can only be obtained approximately, and are expressed by decimals.

6. The *integral* part of any logarithm is called the CHARACTERISTIC, and the *decimal* part is sometimes called the MANTISSA.

7. *The characteristic of the logarithm of* ANY NUMBER GREATER THAN UNITY, *is one less than the number of integral figures in the given number.*

For it has been shown (Art. 3) that the logarithm of 1 is 0, of 10 is 1, of 100 is 2, of 1000 is 3, and so on.

8. *The characteristic of the logarithm of* ANY DECIMAL FRACTION *is a negative number, and is one more than the number of ciphers between the decimal point and the first significant figure.*

For it has been shown (Art. 4) that the logarithm of 0.1 is —1, of 0.01 is —2, of 0.001 is —3, and so on.

NOTE. — In general, whether the given number be integral, fractional, or mixed, *the characteristic of the logarithm of any number expressed decimally is the distance of the first, or left-hand, significant figure from the units' place, being positive when that figure is on the left of the units' place, and negative when on the right.*

GENERAL PROPERTIES OF LOGARITHMS.

9. *The logarithm of a* PRODUCT *is equal to the sum of the logarithms of its factors.*