MATHEMATICAL TRACTS, PART I

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MATHEMATICAL TRACTS.

PART I.

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TRACT I.

ON THE BASES OF GEOMETRY WITH THE GEOMETRICAL TREATMENT OF $\sqrt{-1}$.

CONTENTS.

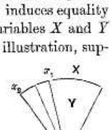
I. THE RATIO OF INCOMMENSURABLES.

1. IN arithmetic the first ideas of ratio and proportion, and the laws of passage from one set of 4 proportionals to another, ought to be learned, as preliminary to geometry; but in geometry the doctrine of incommensurables requires a special treatment, unless the learner be well grounded in the argument of infinite converging series. Repeating decimals may perhaps suffice. Another, possibly better way, is open by the introduction of VARIABLE quantities, which will here be proposed.

2. Nothing is simpler than to imagine some geometrical quantity to vary in shape or size according to some prescribed law. This must imply at least *two* quantities varying *together*. Thus, if an equilateral triangle change the length of its *side*, its *area* also changes. If the radius of a circle increase or diminish, so does the length of the circumference. In general two magnitudes X and Y may *vary together:* they may be either the same in kind,—as the radius and circumference of a circle is each *a length;* or the two may be different in kind, say, a length and an area. In general it is a

convenient notation to suppose that when X changes to X', Y changes to Y'.

3. Again, if X receive successive additions $x_1 x_2 x_3 \ldots x_n$, the corresponding additions (if additions they be) to Y are well denoted by $y_1 y_2 y_3 \ldots y_n$. An obvious and simple case, if it occur, will deserve notice; namely, if the two variables are so regulated, that equality in the first set of additions (i.e. $x_1 = x_2 = x_3 = \ldots = x_n$) induces equality in the second set; (i.e. $y_1 = y_2 = y_3 = \ldots = y_n$). The variables X and Y are then said to *increase uniformly*. As an obvious illustration, suppose X to be the *arc* of a circle, and Y the area of the *sector* which it bounds, evidently then if the arcs x_1, x_2 are equal increments of the arc X, the sectors $y_1 y_2$ which are bounded by $x_1 x_2$ will be equal increments of Y. Then the arc X and the sector Y increase together *uniformly*.



4. We may now establish a theorem highly convenient for application in geometry, alike whether quantities are commensurable or incommensurable.

THEOREM. "If X and Y are any two connected variables, which begin *from zero* together, and increase *uniformly*; then X varies proportionably to Y. In other words, if Y become Y' when X becomes X', then X is to X' as Y is to Y'."

Proof. First, suppose X and X' commensurable, and ξ a common measure, or $X = m \,.\, \xi$ (m times ξ) and $X' = n\xi$. We may then suppose X and X' made up by repeated additions of ξ. Every time that X has the increment ξ, Y will receive a uniform increment which we may call v; then Y is always the same multiple of v that X is of ξ; thus the equation $X = m\xi$ implies $Y = mv$, and $X' = n\xi$ implies $Y' = nv$. Hence $X : X' = m : n = Y : Y'$.

Next, when X' is *not* commensurate with X, yet ξ is some *sub*-multiple of X, such that $n\xi = X$, and X' contains ξ more than m times, but less than $(m+1)$ times; evidently we *cannot* have

$$X : X' = Y : Y'$$

(when the four magnitudes are presented to us) unless, as a first condition, on assuming $nv = Y$, we find Y' to contain v more than m times and less than $(m+1)$ times: and unless this condition were fulfilled, X and Y would *not* increase *uniformly*. We may therefore assume $X_2 X_3$ on opposite sides of X', with values

$$X_2 = m\xi,\ X_3 = (m+1)\,\xi;$$

likewise $Y_2 Y_3$ on opposite sides of Y', with values

$$Y_2 = mv,\ Y_3 = (m+1)\,v.$$

Then by the first case we have $X : X_2 = Y : Y_2$ and $X : X_3 = Y : Y_3$. But $X_3 - X_2 = \xi$, and $Y_3 - Y_2 = v$. Let n perpetually increase, then ξ and v perpetually lessen. X_2 and X_3 run together in X', Y_2 and Y_3 run together in Y'. Thus each of the ratios $X : X_2$ and $X : X_3$ falls into $X : X'$, and each of the ratios $Y : Y_2$, $Y : Y_3$ falls into $Y : Y'$. Inevitably then, $X : X' = Y : Y'$, even when these last are incommensurate. Q.E.D.

II. Primary ideas of the Sphere and Circle.

For the convenience of *beginners*, POSTULATES may be advanced concerning the straight line and the plane, as well as concerning parallel straight lines. But in the second stage of study the whole topic ought to be *treated anew* from the *beginning*: a task which is here assumed.

On Length and Distance.

THEOREM. "All lengths are numerically comparable." To make this clear, it is simplest to imagine a thread indefinitely thin, flexible and inextensible. This, if applied upon any given line, will become an exact measure of its length; and if any two lines be then measured by two threads, the threads are directly comparable, shewing either that they are equal, or that one is longer than the other and how much longer. Hereby we safely assert the same fact concerning any two given lengths.

Obviously, length is *continuous* magnitude: which means, that if a point P run along from A to B, the length AP passes *through all magnitude* from zero to AB.

THEOREM. Any two given points in space may be joined *either* by one path which is shorter than any other possible, *or* by several equal paths than which none other is so short. For of all possible paths joining them some must be needlessly long; yet unless there is some limit to the shortening, the distance would be *nil;* the points would not be two, but would coincide and become one.