INTEREST AND BOND VALUES

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BY

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SECOND EDITION

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PREFACE TO THE FIRST EDITION.

This little book is not to be regarded as a treatise on the Mathematical Theory of Interest—that theory which has been so ably expounded by Mr. King in *The Theory of Finance* and by Mr. Todhunter in the *Text Book of the Institute of Actuaries, Part I.* It is merely an explanation of the Interest Tables and Tables of Bond Values now in common use, and an attempt to instruct students concerning them.

While an elementary knowledge of Algebra is a powerful aid to the intelligent appreciation and use of such tables, and Algebra has not been excluded from the following pages, yet it is believed that nearly everything contained therein may be followed by anyone who will take the trouble to learn the meanings of the standard interest symbols.

Mainly written for the use of the author's own classes in the elementary mathematics of finance, it is hoped that the book may also be of value, not only to actuarial students but also to that increasing number of men who are finding it a business necessity to thoroughly understand the tables referred to

THE UNIVERSITY,
Toronto, January, 1912.

PREFACE TO THE SECOND EDITION.

The author desires to thank many friends for valuable suggestions. He hopes that all errors have been corrected and that in other respects also the book has been rendered more useful especially for classroom purposes.

THE UNIVERSITY,
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INTEREST AND BOND VALUES.

CHAPTER I.

Interest and Discount.

1. We are all aware of the fact that men and corporations of undoubted ability to pay may generally be found who are willing to pay more than a dollar at some future date in return for a dollar today. The excess payment made when the borrowed dollar is returned is called *interest*. We are all equally aware of the corresponding fact that banks and similar institutions will give something less than a dollar today for a good promise to pay a dollar at some future date. The "something less" differs from the dollar by what is called *discount*.

Interest is quoted at so much per cent. per annum, is calculated on the sum lent, and is payable at the end of the year or at the ends of such sub-divisions of the year as may be agreed upon.

Discount is quoted at so much per cent. per annum, is calculated on the sum to be paid in the future, but is itself always payable in advance.

These are facts of common knowledge. Our theory of interest is based on these facts and has nothing whatever to do with the speculations of the Economist who searches for the reasons for these facts.

2. Interest calculations must be as old as civilization. There were money lenders in Thebes and Babylon. Nowadays such calculations commonly occur all over the world. It is therefore not surprising that a world wide system of interest

symbols should have been developed. The elements of this notation are as follows:—

i is the interest on 1 for one period (say a year).

1 at interest for one period will amount to $1+i$.

1 at interest for two periods will amount to $(1+i)^2$.

1 at interest for three periods will amount to $(1+i)^3$.

&c. &c. &c. &c.

1 at interest for n periods will amount to $(1+i)^n$.

Thus if the rate of interest be 5% per annum, $i = .05$ or .05 is the interest on 1 for one year.

1 at interest for one year will amount to 1.05.

1 at interest for two years will amount to $(1.05)^2 = 1.10250$.

1 at interest for three years will amount to $(1.05)^3 = 1.15763$.

1 at interest for n years will amount to $(1.05)^n$.

Again,

v is the present value of 1 due at the end of one period.

v^2 is the present value of 1 due at the end of two periods.

v^3 is the present value of 1 due at the end of three periods.

&c. &c. &c. &c.

v^n is the present value of 1 due at the end of n periods.

3. Since 1 is the present value of $1+i$ due at the end of one period, and v is the present value of 1 due at the end of one period,

therefore $1 : v :: 1+i : 1$, or $v(1+i) = 1$.

$$\text{or } v = \frac{1}{1+i} \text{ and } 1+i = \frac{1}{v}.$$

Thus if the rate of interest be 5% per annum

$$v = \frac{1}{1+i} = \frac{1}{1.05} = .95238, \text{ so that}$$

$v = .95238$ is the present value of 1 due one year hence.

$v^2 = .90703$ is the present value of 1 due two years hence.

$v^3 = .86384$ is the present value of 1 due three years hence.

&c. &c. &c. &c.

$v^n = \frac{1}{(1.05)^n}$ is the present value of 1 due n years hence.

4. *d is the discount on 1 due one period hence*, and obviously d must = 1—v. So we have

$$d = 1-v = 1-\frac{1}{1+i} = \frac{i}{1+i} = iv.$$

Since v at interest for one period amounts to 1, the interest on v for one period must be $1-v$ or d: in short $vi=d$. In other words the present value of the interest on 1 is d. Therefore payments of d in advance each period are equivalent to payments of i in arrears each period.

Again, 1 is the present value of $1+i$ due one period hence.
v is the present value of 1 due one period hence.
$\therefore 1-v=d$ is the present value if i due one period hence.
or $1-d$ is the present value of 1 due one period hence.

5. The three symbols i, d, v, are related as shewn in the following schedule.

The Value of	in terms of		
	i	d	v
i	i	$\frac{d}{1-d}$	$\frac{1-v}{v}$
d	$\frac{i}{1+i}$	d	$1-v$
v	$\frac{1}{1+i}$	$1-d$	v

6. The ordinary interest tables, such as those issued by Colonel Oakes or by Mr. Archer, give the values of $(1+i)^n$ and v^n for numerous values of i ranging from ¼ of 1% up to 10%, and for values of n ranging from 1 period up to 200 periods.

Typical extracts might be:—

n	Amount of 1 at interest for n periods. $(1+i)^n$				n
	1% or $i=.01$	2% or $i=.02$	3% or $i=.03$	4% or $i=.04$	
5	1.05101	1.10408	1.15927	1.21665	5
10	1.10462	1.21899	1.34392	1.48024	10
15	1.16097	1.34587	1.55797	1.80094	15
20	1.22019	1.48595	1.80611	2.19112	20

and

n	Present value of 1 due n periods hence v^n				n
	1% or $i=.01$	2% or $i=.02$	3% or $i=.03$	4% or $i=.04$	
5	.95147	.90573	.86261	.82193	5
10	.90529	.82035	.74409	.67556	10
15	.86135	.74301	.64186	.55526	15
20	.81954	.67297	.55368	.45639	20

7. Although interest is always quoted at so much per cent. per annum, it is usually payable more frequently than once in each year. In interest calculations when interest is payable only once a year it is said to be compounded yearly, or compounded with yearly rests, or to be convertible yearly; but if the interest is payable twice or four times a year it is said to be compounded half yearly or quarterly.

8. It should be noted that many tables use the word "years" in place of the word "periods." This is unfortunate since interest is usually compounded more frequently than once a year, and the word "years" must be understood to mean "half years" or "quarters" as circumstances demand.