NOTES ON ROULETTES AND GLISSETTES

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BY

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PREFACE.

THE following pages contain the explanation of methods, and the investigation of formulæ, which I have for some time past found useful in the discussion of the curves produced by the rolling or sliding of one curve on another.

These methods and formulæ are with a few exceptions original, and, I believe, new; and my object has been to present, from a geometrical point of view, solutions of the various problems connected with Roulettes and Glissettes. I have ventured to introduce, and employ, the word Glissette, as being co-expressive with Roulette, a word which has been in use amongst mathematicians for a considerable time.

The formula of Art. (21) is of course well known; it is given in Salmon's *Higher Plane Curves*, in Walton and Campion's *Solutions*, in Jullien's *Problems*, in Bertrand's *Differential Calculus*, and probably in many other books.

The theorem of Art. (24) was enunciated some years ago, for the particular case of a conic, by Mr Wolstenholme, and extended by myself to the case of any curve. I have however recently found a reference to it in the *Nouvelles Annales* for June, 1869, from which it appears that it was given by Steiner in an early number of the same journal.

For the incisive method of Art. (56) I am indebted to Mr Ferrers.

It will be seen that the general formula of Art. (37) includes most of those which precede it, while it is itself included in that of Art. (55), and that the theorem of Art. (67) reduces all cases of motion in one plane to the cases of Articles (37) or (55).

In a future tract I hope to produce some further developments of the ideas which are here somewhat briefly treated.

W. H. BESANT.

DECEMBER, 1869.

PRELIMINARY REMARKS ON

INFINITESIMALS.

1. An infinitesimal is a quantity which, under certain assigned conditions, vanishes compared with finite quantities.

If two infinitesimals vanish in a finite ratio to each other, they are said to be of the same order.

Thus, if θ vanish, $\sin\theta$ and θ are of the same order, as are also $\sin m\theta$ and $\tan n\theta$.

If two infinitesimals, α and β, be such that the ultimate ratio of β to α^2 is finite, β is said to be of the second order if α be of the first order.

Thus, $1-\cos\theta$, when θ vanishes, is of the second order if θ be of the first order.

And, generally, an infinitesimal which has, ultimately, a finite ratio to the r^{th} power of another is said to be of the r^{th} order if that other be of the first order.

The order of an infinitesimal is, *à priori*, arbitrary and conventional; but, if any standard be fixed upon, the orders of all others are determinate.

2. Consider figure (1), in which O is the centre of a circle, and AP a small arc; PN, PL perpendiculars on OA and on the tangent at A, and Q the point in which OP produced meets AL.

Then, if $OA = a$, and $AOP = \theta$, it can be shewn by Trigonometry that, when θ is indefinitely diminished,

$$\frac{AL}{AP} = 1, \quad \frac{PL}{AP^2} = \frac{1}{2a}, \quad \frac{QL}{AP^3} = \frac{1}{2a^2},$$

and
$$\frac{PQ - PL}{AP^4} = \frac{1}{4a^3}.$$

Therefore, if AP be an infinitesimal of the first order, AL is of the first order, PL of the second, QL of the third, and $PQ - PL$ of the fourth.

3. If α be an infinitesimal of the first order,

$$\lambda\alpha^2 + \mu\alpha^3 : \nu\alpha^2 :: \lambda : \nu, \text{ ultimately,}$$

$$\lambda\alpha^2 + \mu\alpha^3 \text{ is of the second order:}$$

and generally, it will be seen that the order of an infinitesimal is not affected by the addition to it of an infinitesimal of any higher order.

Let AP' be the arc of a curve, such that AP is its circle of curvature at A; then PP' is of the third order, and therefore, so far as quantities of the 2nd order are concerned, P' may be taken to be coincident with P.

4. If AP, AQ be two infinitesimal arcs, of the first order, of two curves touching each other at A, the distance PQ will be of the second order, and therefore, so far as quantities of the first order are concerned, P and Q may be taken to be coincident.

It will be seen that all the preceding theorems are contained in, or deducible from, the 7th and 11th Lemmas of the first section of the *Principia*.

Thus, from Lemma XI., if AP, AP' be two infinitesimal arcs of the same order, and PL, $P'L'$ the corresponding perpendiculars,

$$PL : P'L' :: AP^2 : AP'^2.$$

ROULETTES.

5. WHEN a curve rolls on a fixed curve any given point in the plane of the rolling curve describes a certain curve, which is called a roulette.

Under the same heading we shall also include the curves enveloped by any given lines, straight or curved, which are carried with the rolling curve.

If a curve roll on a fixed curve, the line joining the point of contact with any point Q in the plane of the rolling curve is the normal to the path of Q.

For, as the curve rolls, the point of the curve, P, in contact with the fixed curve, has no motion, and the whole area is, at the instant, turning round P: hence the direction of motion of Q, i.e. the tangent to its path is at right angles to QP, and QP is the normal. (See fig. 2.)

Thus, if a circle roll on a straight line, and if PD be the diameter through P, $Q'P$ is the normal, and $Q'D$ the tangent to the cycloidal path AQ' of a point Q' of the circumference.

6. *A curve rolls on a straight line; it is required to find the roulette traced by any point* Q.

Let the curve roll from O to P, the point A passing over the point O. (See fig. 2.)

Taking O as the origin, and OP as axis of x, let x, y be co-ordinates of Q.