CONIC SECTIONS: THEIR PRINCIPAL PROPERTIES PROVED GEOMETRICALLY

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WILLIAM WHEWELL

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BY

WILLIAM WHEWELL, D.D.,

MASTER OF TRINITY COLLEGE, CAMBRIDGE.

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PREFACE.

I SHOULD not have published the following pages, if it had not been represented to me that there is not at present any geometrical Treatise on Conic Sections, which the Students of the University can conveniently use. Mr. Hustler, in his last edition, has *defined* the Conic Sections by their being Sections of a Cone, cut by a plane in certain manners; and thus he makes it necessary for his readers to begin the subject with Solid Geometry. This is an inconvenience which, I think, we in this University have always endeavoured to avoid, although the method has the sanction of history and antiquity in its favour. The subject is rendered more simple, and better suited to its place as an immediate sequel to Plane Geometry, when we define the figures by their plane properties, as I have done.

The study of Conic Sections in this form appears to be an indispensable part of a course of mathematical reading which goes beyond Elementary Geometry; and especially of a course which includes any study of Newton's *Principia*. The study of Conic Sections as a branch of Analytical Geometry can by no means supply the place of an acquaintance with the geometrical proofs. By seeing the subject presented only in an analytical form, the student obtains a very erroneous notion of the relation of the subject to geometry, and remains utterly ignorant of that part of mathematics which all mathematicians up to the present time have understood by the term *Conic Sections*. Nor can he acquire a proper apprehension of the peculiar beauty and generality of the language of Analytical Geometry, without seeing the properties of the Conic Sections *before* they are translated into that language. Moreover it is not too much to say that those students who have never become acquainted with the reasonings of Conic Sections in a geometrical form, will unavoidably have a very confused and imperfect comprehension of the reasoning of the *Principia* of Newton; and cannot help looking upon the study of that work as a troublesome and incongruous excrescence in their mathematical course.

I am fully aware that those students of mathematics who proceed to the higher parts of the science must give a great portion of time to analytical studies: and that, in the common course of mathematical reading, no large space can be assigned to the subject here treated of. On that account, I have made the Treatise very brief, and have inserted none but what appear to be *classical* Propositions in Conic Sections. Nor indeed have I introduced all such Propositions. The mathematical reader who is acquainted with the subject, will here miss many properties of great beauty, which he will have seen elsewhere; and which I have sacrificed to the brevity which I thought desirable. For the same reason, when some properties of the Ellipse have been proved, I have omitted the analogous properties of the Hyperbola (as Props. XVI, XVII. of the Ellipse). The reader will have no difficulty in supplying the proof from the analogies of the other properties. For like reasons, the steps of Elementary Geometry which occur in the proofs are given in a very concise manner. The geometrical student will be able to unfold the reasoning more fully; and it will be an instructive exercise to do so.

In reference to the Propositions and their proofs, I may observe that I have thought it desirable to prove, for each curve, first, and independently, those properties which do *not* depend upon a *Tangent* to the curve; as the properties with reference to the Principal Axis. I have not attempted to give *analogous* proofs of the analogous Propositions of the Ellipse and Hyperbola with regard to their Diameters; (Ellipse, Prop. XII, and Hyperbola, Prop. XIV;) for the proofs are simplified in the two cases by relations special to each; the relations of the Ellipse to the Circumscribing Circle, and of the Hyperbola to its Asymptotes. In the other Propositions, the analogy of the demonstrations is preserved.

TRINITY LODGE,
May 13, 1846.

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INTRODUCTION.

(1) By calling proofs *geometrical*, we mean that they are deduced from the propositions in Euclid's Elements of Geometry, by reasoning of the same kind as that which is used in the Elements.

But though this is the method pursued in the following pages, I have employed some modes of expression which are not used in the common editions of Euclid; and have also assumed a few simple geometrical propositions which do not stand in the Elements. I will enumerate the principal points of this kind.

(2) I have used the algebraical signs $=$, $+$ and $-$. These are to be understood as being only marks for words, not symbols by means of which operations are performed.

Thus $A + B$ denotes "A together with B;" $A - B$ may be read "A, its excess over B." And in order to preserve the geometrical character of the reasoning, we must not apply indiscriminately the algebraical rule of transposing terms and changing their signs. Thus, if $A - B = C - D$, we must not infer at once that $A + D = C + B$. But we may deduce this result by two steps, thus: $A - B + D = C$, by adding D to the equals: and hence, $A + D = C + B$, by adding B to the equals. This is not a merely fanciful distinction; for most minds moderately familiar with geometrical reasoning, can follow one of these steps at a time, but not both together.

If $A - B = 2P$, and $A + B = 2Q$; then $A = P + Q$, $B = Q - P$.

(3) Also $A \cdot B$ or AB is used to express the rectangle of lines A and B, and A^2 to express the square of A. And the propositions of Euclid's Second and other Books are expressed by means of this notation. For instance, Eucl. II. 3, is thus expressed; $A(B+A) = AB + A^2$; and Eucl. II. 4, thus; $(A+B)^2 = A^2 + B^2 + 2AB$. Also Eucl. II. 5, Cor. is thus expressed; $A^2 - B^2 = (A+B)(A-B)$. And to express Eucl. II. 7; if A be the whole line, B one of the parts, $A - B$ the other part; we have $A^2 + B^2 = 2AB + (A-B)^2$; and hence taking away $2AB$ from both,

$$A^2 + B^2 - 2AB = (A-B)^2,$$

which proposition we shall also assume.

(4) Proportion is expressed in the usual manner,

$$A : B :: C : D.$$

And the propositions of Euclid's Fifth Book (including Simson's) are represented accordingly. Thus

if $A : B :: C : D$,

Eucl. V. B, $B : A :: D : C$; (*invertendo*).

Also Eucl. V. 16, $A : C :: B : D$; (*alternando*).

Also Eucl. V. 17, $A : A - B :: C : C - D$; (*dividendo*).

Also Eucl. V. 18, $A + B : B :: C + D : D$ (*componendo*).

Also Eucl. VI. 16, if $AD = BC$, $A : B :: C : D$.

Other consequences of the proposition $A : B :: C : D$ are also assumed, and might easily be proved; namely,

$$A + B : A - B :: C + D : C - D;$$

$$A - B : B :: C - D : D;$$

$$A^2 : B^2 :: C^2 : D^2;$$

$AM : BM :: CN : DN$; and the like.

(5) The composition of proportions by algebraical multiplication may be justified by geometrical reasoning; and is here assumed. Thus

if $A : B :: C : D$,

and $M : N :: D : E$,

and $N : A :: P : C$;

we infer that $M : B :: P : E$ (Ellipse, Prop. XI.);

which we may thus prove. The composition of the two first proportions gives us, by Eucl. VI. 14, and V. 22,

$$AM : BN :: C : E,$$

and the third proportion gives us

$$MN : AM :: P : C;$$

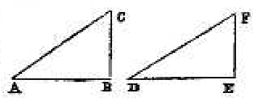
whence by Eucl. V. 22,

$$MN : BN :: P : E;$$

that is, $M : B :: P : E$.

The following Propositions may be assumed.

(6) If ABC, DEF be two triangles, right-angled at B and E, and having the sides AB, AC equal respectively to DE, DF; the triangles are equal in all respects.



For $BC^2 = AC^2 - AB^2$, and $EF^2 = DF^2 - DE^2$; whence $BC^2 = EF^2$; and $BC = EF$; and hence, by Eucl. I. 8, the triangles are equal in all respects.

(7) In any triangle, if the base be divided into segments by a perpendicular from the opposite angle;

base : sum of sides :: diff. of sides : diff. of segments.

Let HPS be a triangle, PM the perpendicular falling upon HS in M; then $HM^2 + MP^2 = HP^2$; and $SM^2 + MP^2 = SP^2$, by Eucl. I. 47.

Hence $HM^2 - SM^2 = HP^2 - SP^2$.

Hence, $(HM + SM)(HM - SM)$

$= (HP + SP)(HP - SP)$,

by Eucl. II. 5.