PARTIAL DIFFERENTIAL EQUATIONS. AN ESSAY TOWARDS AN ENTIRELY NEW METHOD OF INTEGRATING THEM

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Partial Differential Equations. An Essay Towards an Entirely New Method of Integrating Them by S. Earnshaw

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AN ESSAY

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BY

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OF ST JOHN'S COLLEGE, CAMBRIDGE.

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PREFACE.

THE Author of this Essay has been forewarned that the adoption of the subscript differential notation, which forty years ago was in some degree of favour but afterwards laid aside, will prove a serious hinderance to the acceptableness of the present Work, as it will be new to the generality of mathematical students, and therefore somewhat repellent. He regrets the circumstance, and feels that an explanation and apology are needed; and he finds both in the fact that the notation alluded to has been forced upon him by the necessities of the case; and, moreover, he ventures to think that the reader after going through the Essay will agree with him in this, and be willing on this ground to excuse such an important deviation from established mathematical usage.

The Author denominates his Work an ESSAY, because as a TREATISE it would be very incomplete, there being many portions of the general subject not alluded to in these pages. For information on those portions he must refer the reader to Boole and Gregory and Carmichael, with whose admirable and valuable Works he has been careful to interfere as little

as possible. In fact, he has consulted them only for such illustrative examples as fell within the scope of his Essay. He is conscious of having apparently intermeddled with the integration of equations of *one* independent variable; but this has arisen out of the peculiarities of the Author's system, by which every equation, whatever be the number of its independent variables, is reduced to an equivalent equation of only one independent variable. It was therefore impossible to keep the two branches of differential equations apart.

The system of integration here proposed occurred to the mind of the Author a few years ago, but his professional engagements did not then leave him leisure to follow the general idea into its details. And, as his object is merely to render his Method thoroughly intelligible, and not the exhibition of integrals, he has for brevity's sake not always proceeded to the last steps of an integration when he conceived that the method had been made intelligible.

The Author fears a cursory glance at the pages of the Work will have a prejudicial effect; for he is aware that some of them exhibit a formidable and deterrent array of novel symbols; he therefore begs to assure the reader that the various steps of the investigations are all obedient to one general principle, and though in some degree novel, are not really difficult, but on the contrary easy when the eye has become accustomed to the novelties of the notation. And, moreover, he entertains a hope that the results of integrations (many of which are far more general than they were in the shape in which they have appeared in former Treatises) will repay the reader for any extra trouble he may find in pushing through what may at first appear to

him a forest of operative symbols. Many equations will be found in this Essay integrated with ease in finite terms, which, as far as the Author is aware, were never integrated in finite terms before. In the last Chapter, for instance, the following highly important equations will be found integrated for the first time in finite terms:

$$\frac{d^2u}{dx\,dy}+cu=0;$$

$$\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+cu=0;$$

and

$$\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}=0.$$

Three more important partial differential equations could hardly be instanced, and a finite integral of each one of them is obtained in this Essay without any difficulty.

The Author can scarcely hope that in a work so novel in its methods as this is, and written as this has been under considerable pressure of other engagements, some clerical errors will not have escaped his notice; but he hopes that they will not prove to be many, and that none of them will involve errors of principle. If any are found, he will be thankful if the courteous reader will send notice of them addressed to him at the Publishers.

SHEFFIELD,
October 2, 1871.