NUMBERS, AND HOW TO USE THEM: BY THE NATURAL METHOD: A COMPLETE, PRACTICAL ARITHMETIC, FOR SELF-HELP AND FOR SCHOOLS OTHER THAN THE PRIMARY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649536825

Numbers, and How to Use Them: By the Natural Method: A Complete, Practical Arithmetic, for Self-Help and for Schools Other than the Primary by John F. Brown

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

JOHN F. BROWN

NUMBERS, AND HOW TO USE THEM: BY THE NATURAL METHOD: A COMPLETE, PRACTICAL ARITHMETIC, FOR SELF-HELP AND FOR SCHOOLS OTHER THAN THE PRIMARY

Trieste

NUMBERS,

AND HOW TO USE THEM:

BY THE NATURAL METHOD.

A COMPLETE, PRACTICAL ARITHMETIC, FOR SELF-HELP AND FOR SCHOOLS OTHER THAN THE PRIMARY.

BY

JOHN F. BROWN.

Boston, MASS.:
PUBLISHED BY JOHN F. BROWN,
304 Washington St.

MAILED, POSTPAID, FOR $1.00.

Typography by J. S. Cushing & Co., Boston.

INTRODUCTION.

THE NATURAL METHOD, as I have chosen to term it, of presenting the Science of Arithmetic, differs from methods now and formerly employed in four important respects:

First and foremost, numbers are treated as always abstract.

There is nothing novel in this *conception* of number. Webster says number in mathematics is "that abstract species of quantity which is capable of being expressed by figures." And again, pure mathematics "considers magnitude or quantity abstractly, without relation to matter." The vocabulary of "A Practical Arithmetic," by Wentworth and Hill, has the following explanations:

"**Abstract number**. — This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned*."

"**Concrete number**. — A phrase without meaning. Things numbered are concrete, but the number is abstract."

Second, all operations upon numbers are deduced from the fundamental process of counting.

Third, formal definitions and rules are altogether omitted.

Fourth, geometrical principles are reserved for the science of geometry, where they properly belong.

That geometry has no place in an arithmetic ought to go without saying. Yet the practice has always been otherwise. Arithmetics are filled with geometrical demonstrations, definitions, and problems. The result is a vagueness and confusion of ideas, which, in the great majority of cases, is never outgrown. For instance, much of the perplexity that exists in regard to fractions is directly traceable to this failure to discriminate between the geometrical and the arithmetical. An apple may be cut into equal pieces, or a line into equal parts; but, the foundation principle of number being mere consecutiveness, a piece of a numerical unit is utterly incomprehensible. Yet a piece of the apple and parts of the line are employed as symbols to illustrate $\frac{1}{2}$ and $\frac{2}{3}$; and numerical fractions, never having been taught, are never comprehended.

For an analogous reason, subjects requiring for a simple and satisfactory treatment an extension of the conditions and operations of arithmetic ~~the use of equations, or of formulæ, to any considerable extent~~, are relegated to algebra. President Eliot says he was never so indignant as when he came to Harvard at fifteen and found that two-thirds of the problems in arithmetic, which he had been fighting over for years, could be solved in a few hours by algebra. Such problems have no place in an arithmetic. In fact, if one thoroughly understands pure numbers and the general principles of applying them, he will find little difficulty with anything that properly comes within the scope of arithmetic.

By thus simplifying arithmetic, geometry and algebra may be taught much earlier than they are at present, and students well grounded in the principles of extension and magnitude and in the use of equations and formulæ, at an early age.

This book treats of arithmetic only. But the ideal method of teaching mathematics, so far as it is commonly taught in the public schools, so far as people in general may be expected to know anything about it, is to treat the several branches in connection under one general head. It may all be put into one moderate sized volume, no larger than the present common school arithmetics, no more difficult of comprehension, and requiring no more time for its mastery. According to this plan, after fractions, number would be applied to various kinds of units (dollars, bushels, pounds, a, b, x, y, etc.), the principles of arithmetic being generalized by means of algebraic symbols and formulæ. After this would come powers and roots, arithmetical and algebraic; then, the fundamental principles of geometry, beginning almost immediately with the development and measurement of lines, surfaces, and solids. Percentage and interest, arithmetical and geometrical progressions, and other topics, would follow in due course, the general scheme allowing very great condensation and simplicity of treatment.

If the reception of the present volume should be such as to indicate that the times are ripe for the more comprehensive work above outlined, *Mathematics for Common Schools and General Use*, comprising arithmetic, algebra, plane and solid geometry, may appear later.

The old idea was that "a definition is a basis for thought and reasoning." "A child should not be expected to frame a good definition," and therefore it must be framed for him. Of late the tendency has been "to avoid a multiplicity of rules" and definitions. Still the reform has been but an indifferent one; the same fault exists as formerly, though in a lesser degree. If the pupil cannot frame a good definition after a subject has been explained, the reason is because he is not as yet sufficiently advanced, and for this very reason the definition should be deferred. The main purpose of scientific definitions, in the language of John Stuart Mill, "is to serve as the landmarks of scientific classification." We cannot classify things correctly unless we first have a thorough and complete knowledge of the things under consideration, and until then we should avoid definitions as much as possible.

To illustrate: having first given a simple example of addition, it is safer to say *This is Addition*, than it is to say *Addition is this or that*, or *This or that is Addition*. Here are two definitions of addition, taken respectively from two text-books now largely used in the public schools of Massachusetts: "*Addition* is the process of uniting two or more *like* numbers into *one* equivalent number." "The process of counting numbers together is *Addition*." What idea is conveyed by these definitions? How do we *unite* 2 and 3 to make 5? Or how do we *count* 2 and 3 *together* to make 5? Unless we conjure up a false and unnatural theory of "ideas in the mind" to "take the place of objects without," thus dealing with number in the abstract by referring it to objects, when in reality it is only to ideas and not to objects at all that number as a mathematical term applies, these things are unintelligible as strict and literal expressions.

Taken metaphorically, as mere likenings of one thing to something else these quasi-definitions may do very well. But the trouble is, a true definition is much more than this. It illustrates the *connotation* of the term defined; that is, it tells what the term *means*. And how are we to know whether that which reads like a definition is intended to be a real definition or merely a description, more or less complete? It is likely to be taken in the former sense whether it fulfils the requirements or not.

Definitions are of two general classes, those that are comprehensive and exact, and those that are not. The former are the exception, the latter the rule. The mischief wrought by definitions is threefold: first, being usually inexact, they tend to create false ideas; second, it is so much easier to repeat phrases than it is to think, that mere memorizing will be indulged in when so favorable an opportunity is presented; third, they tend to divert the mind from that with which it should be occupied to abstruse and metaphysical questions which ought to be kept in the background. These remarks are intended to apply only to cases where the subject under consideration is abstruse, like that of number.

And of what value are rules? The pupil is first shown how to do a problem, and then is told how to do it. In other words, general principles are applied to a particular problem, a rule is deduced, and afterwards problems coming under the rule are performed by referring them to the rule and following the directions there given. Thus, the pupil is trained to follow rules. He recites rules, he works by rule. If he has occasion to apply the same general principles in a different manner, not having been trained to work without rules, nor even to make rules of his own, but to look for them ready made, the chances are that he is unable to make the application.

Rules, definitions, and the absurd *analysis* of problems which has been so long in vogue, certainly conduce to glib recitations; but where that is made the desideratum, proficiency in figuring need not be expected.

After becoming well grounded in the science, the framing of definitions *by the student* may be a very good exercise. And so, after fundamental principles are understood, and the methods of applying them in general, special modes of operating may perhaps be advantageously expressed in the form of rules.

That all operations upon numbers are reducible to one is no new principle, but, so far as I am aware, the principle has never before been carried out to its logical completeness by *de*ducing all operations *from* one. It is not because the desirability of so simple and logical a scheme has not been appreciated, but because it has been thought there were insuperable obstacles in the way, as indeed there have been with "concrete numbers" as the starting-point. And whatever the writers may say to the contrary, all previous arithmetics are tainted with the concrete fallacy, as evidenced by their treatment of division and fractions.

In answer to the question, — "From your experience as a teacher, what do you say as to the proficiency in the use of figures of pupils commencing the high school course, after completion of the course in the lower grades?" — a well-known instructor writes:

"I must with sorrow say that from year to year the pupils of our high school enter with less and less ability to perform the ordinary calculations in arithmetic. I am unable to get them to use any reason in the solution of a problem."

Another teacher of note, in a different part of the state, replies in answer to the same question:

"I have 129 pupils in my Class IV, nearly all of whom left our grammar schools last June. There they had studied arithmetic for a number of years. On entering this school they have a weekly lesson in arithmetic, partly review and partly advance, for four years more. This class had been at work reviewing percentage for eight weeks when I examined the weekly marks in arithmetic with which my assistants sup-