A TREATISE ON THE CALCULUS OF OPERATIONS: DESIGNED TO FACILITATE THE PROCESSES OF THE DIFFERENTIAL AND INTEGRAL CALCULUS AND THE CALCULUS OF FINITE DIFFERENCES

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A treatise on the calculus of operations: designed to facilitate the processes of the differential and integral calculus and the calculus of finite differences by Robert Carmichael

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ROBERT CARMICHAEL

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Trieste

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DIFFERENTIAL AND INTEGRAL CALCULUS

AND THE

CALCULUS OF FINITE DIFFERENCES.

BY THE

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TO

THE BOARD OF TRINITY COLLEGE, DUBLIN,

BY WHOSE KIND LIBERALITY

THE EXPENSES OF PUBLICATION HAVE BEEN MAINLY DEFRAYED,

This Work

IS GRATEFULLY DEDICATED BY

THE AUTHOR.

PREFACE.

THE want of a text-book on the Calculus of Operations has long been felt by mathematicians. The extensive practical bearings of the Differential and Integral Calculus, and the theoretic interest which is associated with the Calculus of Finite Differences, render it desirable that the processes required in these branches of analysis should be reduced and simplified as far as possible. To the student, the Calculus of Operations proposes to facilitate and abbreviate his labours, while, to the advanced mathematician, it offers a method which will enable him not only to arrive at known results with ease, and express them with elegance, but also to extend his investigations with certainty and rapidity.

To illustrate, however inadequately, the power of this Calculus, is the object of the following Treatise. In its preparation all prolixity of detail has been stu-

diously avoided, as alike unnecessary and wearisome. It has been sought also to exclude, as far as possible, metaphysical subtleties, which might perhaps lend an air of learned mystery, but which serve only to embarrass the reader and weaken his confidence in the results at which he may have arrived.

With a view to the partial indication of the nature of the subjects discussed in the Fellowship Lectures of this University, and more particularly in so far as the development of this branch of analysis is concerned, I have been requested to append notes to the various articles derived from this source. For greater facility of reference, and as a contribution towards the history of this department of science, I have been induced to adopt the same course with regard to assistance derived from other sources. Wherever the subject of any article has been originated by another, and the investigation or method of treatment is, so far as I am at present aware, my own, the reference is appended immediately after the statement of the subject of the article. Wherever the subject and method of treatment are both due to another, the reference is given at the end of the article. It is not, of course, necessary, nor would it, indeed, be possible, to extend this system of reference to the case of those articles which will be at once recognised as common property.

Where any of the results contained in the following pages have been already published by myself, either in the "Cambridge and Dublin Mathematical Journal," or in the "Philosophical Magazine," I have in such cases simply stated the name of the periodical in which such results may have appeared. With the exceptions stated, the remainder of the book is, I believe, new.

My first and largest acknowledgments are due to the Rev. John Hewitt Jellett, Professor of Natural Philosophy in this University, whose Treatise on the "Calculus of Variations" first led me to independent and original investigation. My next acknowledgments are due to the Rev. Charles Graves, Professor of Mathematics in this University, whose investigations in this branch of analysis have largely contributed to illustrate its elegance and power, as the following pages will abundantly testify, and to whom I am indebted for acquaintance with many valuable sources of information. The amount of assistance which I have derived from the valuable collection of Examples illustrative of the processes of the Differential and Integral Calculus by the late Mr. Gregory, Fellow of Trinity College, Cambridge, is very considerable, and much of the importance now attributed to the Calculus of Operations is due to the vindication and illustration of its claims by that distinguished mathematician. My

acknowledgments are also due to Sir John Herschel, whose Supplement to the translation of "Lacroix's Differential and Integral Calculus," so remarkable for the subtlety of its reasonings and the breadth of its conceptions, I have studied with much advantage.

I would offer my best thanks to Mr. Arthur Curtis, to whom I am indebted for many valuable suggestions and much kind assistance in the revision of the sheets of this work during its progress through the press.

In dedicating my book to the Board of Trinity College, I have endeavoured to show my appreciation and respect for the enlightened liberality with which they invariably support every genuine effort for the advancement of learning.

TRINITY COLLEGE, DUBLIN,
March, 1855.

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