CURVE TRACING IN CARTESIAN COORDINATES

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649022816

Curve tracing in Cartesian coordinates by William Woolsey Johnson

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

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IN

CARTESIAN COORDINATES

BY

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London
MACMILLAN & CO
1885

PREFACE

THIS book relates, not to the general theory of curves, but to the definite problem of ascertaining the form of a curve given by its equation in Cartesian coordinates, in such cases as are likely to arise in the actual applications of Analytical Geometry. The methods employed are exclusively algebraic, no knowledge of the Differential Calculus on the part of the reader being assumed.

I have endeavored to make the treatment of the subject thus restricted complete in all essential points, without exceeding such limits as its importance would seem to justify. This it has seemed to me possible to do by introducing at an early stage the device of the Analytical Triangle, and using it in connection with all the methods of approximation.

In constructing the triangle, which is essentially Newton's parallelogram, I have adopted Cramer's method of representing the possible terms by points, with a distinguishing mark to indicate the actual presence of the term in the equation. These points were regarded by Cramer as marking the centres of the squares in which, in Newton's parallelogram, the values of the terms were to be inscribed; but I have followed the usual practice, first suggested, I believe, by Frost, of regarding them merely as points referred to the sides of the triangle as coordinate axes. It has, however, been thought best to return to Newton's arrangement, in which these analytical axes are in the usual position of coordinate axes, instead of placing the third side of the triangle, like De Gua and Cramer, in a horizontal position.

The third side of the Analytical Triangle bears the same relation to the geometrical conception of the line at infinity that the other sides bear to the coordinate axes. I have aimed to bring out this connection in such a way that the student who desires to take up the general theory of curves may gain a clear view of this conception, and be prepared to pass readily from the Cartesian system of coordinates, in which one of the fundamental lines is the line at infinity, to the generalized system, in which all three fundamental lines are taken at pleasure.

Lists of examples for practice will be found at the end of each section. These examples have been selected from various sources, and classified in accordance with the subjects of the several sections.

W. W. J.

U. S. Naval Academy,
November, 1884.

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CURVE TRACING

I

Equations Solved for One Variable

1. THE equation of a curve is in these pages supposed to be given in Cartesian coordinates; and the curve is said to be *traced*, when the general form of its several parts or branches is determined, and the position of those which are unlimited in extent is indicated. In the diagrams the coordinate axes will, for convenience, be assumed rectangular; but the methods are equally applicable to oblique axes. When it is possible to solve the equation for one of the coordinates, so as to express its value in terms of the other, the resulting form of the equation affords the most obvious method of tracing the curve. This may be done for either variable when the equation is of the second degree in both variables, in which case the curve is a *conic*. For example, let the given equation be

$$2x^2 - 2xy + y^2 - 4x + 2y + 1 = 0. \qquad (1)$$

Solving for y, we have

$$y = x - 1 \pm \sqrt{(2x - x^2)}. \qquad (2)$$

Thus, for any given value of x, we have two values of y; and if we put

$$y' = x - 1, \tag{3}$$

the equation of the curve becomes

$$y = y' \pm \sqrt{(2x - x^2)}. \tag{4}$$

Diameters

2. Equation (3) represents a straight line, and equation (4) shows that the two ordinates for the curve may be found by adding to and subtracting from the ordinate of the straight line the same quantity. In other words, the chord joining the two points of the curve which have the same abscissa, that is, any chord parallel to the axis of y, is bisected by the straight line. This line is therefore called a *diameter* of the curve. The diameter represented by equation (3) is constructed in Fig. 1.

3. The radical $\sqrt{(2x - x^2)}$, which is half the length of the chord, varies with x, and vanishes for the two values

$$x = 0 \quad \text{and} \quad x = 2;$$

the corresponding points on the diameter are therefore also points of the curve. Writing the radical in the form

$$\sqrt{[x(2 - x)]},$$

it is obvious that all values of x between 0 and 2 give real values to the radical, since they render both of the factors