

**THE PRINCIPLES OF THE DIFFERENTIAL
AND INTEGRAL CALCULUS, SIMPLIFIED,
AND APPLIED
TO THE SOLUTION OF VARIOUS USEFUL
PROBLEMS IN PRACTICAL
MATHEMATICS AND MECHANICS**

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The Principles of the Differential and Integral Calculus, Simplified, and Applied to the Solution of Various Useful Problems in Practical Mathematics and Mechanics by Thomas Tate

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BY
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TO
THE REV. H. MOSELEY, F.R.S.
&c. &c.

ONE OF HER MAJESTY'S INSPECTORS OF SCHOOLS.

REVEREND AND DEAR SIR,

This work I inscribe to you, as a tribute of esteem, on account of the discoveries with which you have enriched physical science; and as an acknowledgment of the value which I attach to your services in promoting elementary education.

That you may long continue to be a distinguished instrument, under the blessing of Divine Providence, in elevating the intellectual and moral condition of the people of this country, is the earnest wish of

Your humble and obliged servant,

THOMAS TATE.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 841. 842. 843. 844. 845. 846. 847. 848. 849. 850. 851. 852. 853. 854. 855. 856. 857. 858. 859. 860. 861. 862. 863. 864. 865. 866. 867. 868. 869. 870. 871. 872. 873. 874. 875. 876. 877. 878. 879. 880. 881. 882. 883. 884. 885. 886. 887. 888. 889. 890. 891. 892. 893. 894. 895. 896. 897. 898. 899. 900. 901. 902. 903. 904. 905. 906. 907. 908. 909. 910. 911. 912. 913. 914. 915. 916. 917. 918. 919. 920. 921. 922. 923. 924. 925. 926. 927. 928. 929. 930. 931. 932. 933. 934. 935. 936. 937. 938. 939. 940. 941. 942. 943. 944. 945. 946. 947. 948. 949. 950. 951. 952. 953. 954. 955. 956. 957. 958. 959. 960. 961. 962. 963. 964. 965. 966. 967. 968. 969. 970. 971. 972. 973. 974. 975. 976. 977. 978. 979. 980. 981. 982. 983. 984. 985. 986. 987. 988. 989. 990. 991. 992. 993. 994. 995. 996. 997. 998. 999. 1000.

PREFACE.

THE object of this work is to explain, illustrate, and apply the fundamental principles of the Calculus, in such a manner as to bring them within the comprehension of a student, having merely a knowledge of ordinary Algebra and Trigonometry, and to enable him to undertake the perusal of such valuable practical works as "Moseley's Principles of Engineering," "Navier's *L'Application de la Mécanique*," "Whewell's *Mechanics*," "Hann's *Treatise on the Steam Engine*," &c.

To a person merely acquainted with ordinary algebra, the Calculus must, at first, appear mysterious and metaphysical; for he has to view abstract quantities, not only in an isolated form, but as admitting of continuous changes, and of taking certain finite ratios as they approach zero or infinity. The principle involved in a limiting ratio must, however, be eventually understood by every student who wishes to make a satisfactory progress in this branch of analysis. I have adopted the method of limits almost exclusively in this work, because it appears to be the most natural and consistent foundation of the Calculus; and with the view to simplify this method as much as possible, I have fully explained and applied it in the preliminary portion of this treatise, apart from the conventional and abstract notation by which the condition of a limit is usually expressed.

It is highly desirable that Teachers and Practical Men should possess some knowledge of this most important branch of pure mathematics, in order to enable them to understand

our best works on mechanical and experimental philosophy. The great physical laws, by which it has pleased the Almighty to govern the universe, must always form a lofty subject of contemplation to his intelligent creatures; but these laws can only be duly interpreted by the aid of the symbolic language of the higher analysis.

As a complete knowledge of a great subject, like this, cannot be obtained from the perusal of one book, those who aspire to a further acquaintance with the higher parts of the Calculus must study the works of De Morgan*, Moigno, O'Brien, Hymers, Gregory, Price†, Hall, Moseley‡, and Young.

T. TATE.

Battersea, Feb. 1849.

* "De Morgan's Calculus" is the largest work on the subject in our language.

† On the method of Infinitesimals.

‡ On Definite Integrals, published in the "Encyclopædia Metropolitana."

TABLE OF CONTENTS.

| | PAGE |
|---|------|
| INTRODUCTORY PRINCIPLES, &c. - - - - - | 1 |
| The Binomial Theorem - - - - - | 2 |
| Indeterminate Coefficients - - - - - | 6 |
| Equations to Curves - - - - - | 11 |
| On the limiting Values of Quantities - - - - - | 22 |
| Illustrations and Applications - - - - - | 23 |
| Increments and their limiting Ratio - - - - - | 56 |
| DIFFERENTIAL CALCULUS. Notation, &c. - - - - - | 40 |
| Rules for the Differentiation of Functions, with Examples, &c. - - - - - | 48 |
| Maxima and Minima - - - - - | 57 |
| Examples of Maxima and Minima - - - - - | 60 |
| Rules for Differentiation continued, with Examples, &c. - - - - - | 75 |
| Successive Differentiation - - - - - | 92 |
| Maclaurin's Theorem. Applications - - - - - | 94 |
| Taylor's Theorem. Applications - - - - - | 101 |
| Vanishing Fractions - - - - - | 107 |
| Tangents to Curves - - - - - | 110 |
| Asymptotes to Curves - - - - - | 114 |
| Equation to the Tangent, &c. Examples - - - - - | 116 |
| Points of contrary Flexure - - - - - | 121 |
| Tracing of Curves - - - - - | 123 |
| INTEGRAL CALCULUS - - - - - | 127 |
| Elementary Rules of Integration, with Examples - - - - - | 128 |
| Fundamental Formulæ - - - - - | 139 |
| Reduction of Integrals, by easy Algebraic processes, to fundamental forms - - - - - | 140 |
| Integration of Rational Fractions - - - - - | 146 |
| Integration by the method of Parts - - - - - | 154 |
| Method of Rationalisation - - - - - | 158 |
| Method of Reduction - - - - - | 161 |
| Integration of Exponential and Logarithmic Functions - - - - - | 170 |