THE ELEMENTARY GEOMETRY OF THE RIGHT LINE AND CIRCLE, FOR THE USE OF SCHOOLS AND COLLEGES, WITH EXERCISES

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BY

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PREFACE.

THIS treatise is published with a view to facilitate the learning of Geometry in its first stages. It is very generally acknowledged that Euclid's "Elements" is unfit, as a text-book, for the quick and effective education in Geometry of the youth of the present age. A treatise which would combine all the excellences of Euclid with the advantages of the nomenclature and methods of Modern Geometry, and place the subject before the student in a plain, natural order of development, has been very much desired. This work professes to be, at least, a fair approximation to the ideal of such a handbook. It, no doubt, has its faults; but, taken as a whole, it may, nevertheless, be a move in the right direction.

The course I have followed is, to take the materials of Euclid, to add to them, and, retaining his plan of stating every distinct theorem or problem in a formal proposition, to arrange the whole in what I should conceive to be a simple and natural order of demonstration—such an order as was suggested, two hundred years ago, by the distinguished Antoine Arnauld, in his "Port-Royal Logic"—and to do this, not in an antiquated and semi-syllogistic style, but in the ordinary language in which all modern English mathematical books are written.

The first step was to separate the problems from the theorems, on the ground that the former are by no means necessary to the demonstration of the latter. The reasons for taking this course are given in the Second Note to the Appendix, the chief of which is, that problems not only derange the natural order of the theorems, but unnecessarily do so. Consequently, the problems in this treatise are given at the ends of the several chapters, excepting the first, which hardly furnishes material for a problem, being confined to the right line, or directive.

The course was thus open for a natural arrangement of the subject. After much consideration, and having in mind the acknowledged fact—"The reproach of geometry"—that Euclid never proved the twenty-ninth proposition of his First Book on any basis of postulation or assumption more evident than the proposition itself, I could not resist the conclusion that there was, in his system, some definition left out or important geometric idea overlooked. That idea was Direction. (*See Appendix, Note* 1.) Afterwards, I ascertained that "direction" had often been suggested by geometricians — as may be seen in the article on Parallels in the Penny Cyclopædia—as the true basis of a doctrine of parallel lines. Confirmed thus in my convictions, it became clearly evident to me that right lines, or directives, must contain within themselves, in their intersections, all their angular properties without any need of finite line, triangle, or circle, to make them manifest. Hence, the

First Chapter of this treatise is confined to the consideration of the directive, the simplest of all figures—if it may be called a "figure"—the distinctive quality of which is *sameness* of direction throughout.

The next, and only other, figure of which Elementary Geometry treats is the Circle, the natural function of which seems to be the furnishing of a comparative measure of the lengths of finite right lines, that is, of portions of directives—which be equal, which greater, and which less? Thus the directive should precede the circle; and the circle, which by itself has no property but symmetry of form, in its intersections with directives, and aided by the directive, should make manifest its own peculiar properties. This seems to me to be the natural and truly philosophical basis of an elementary geometry of the right line and circle; and, for that reason, the Second Chapter is given to the circle, which is first considered in reference to its intersections with directives and with other circles, and afterwards as to its tangencies.

The properties of directives and circles being thus established, the next step is to apply them to the determining of the relations of the sides and angles of a triangle to each other, and to the several cases of equal triangles—also to the relations of the sides and angles of parallelograms, which are but cases of two equal triangles on opposite sides of a diagonal. The isosceles triangle, the properties of which come immediately from the circle, takes the lead in these demonstrations, which are given in the Third Chapter.

In these three chapters there is no mention of *an area*. They are confined to the right line and circle in their relations of angular and linear magnitudes, including those of the triangle and parallelogram. Area is, for the first time, introduced in the Fourth Chapter, in which it is considered, by the aid of Euclid's principle of superposition, in reference to areas *generally*, to the equality of the areas in the several cases of equal triangles, and of equal segments and sectors of circle—and, also, the equality of the areas of parallelograms and triangles on the same or equal bases and between the same parallels. Thus, in the first four chapters, almost the whole substance, theorems and problems, of Euclid's first, third, and fourth books are given.

Euclid's Second Book, placed between two comparatively easy ones, is out of place, and has been a sore stumbling-block to young beginners. Its subject-matter is here held back for the Fifth Chapter, not on account of this difficulty, but because the theorems and problems are more advanced, and should, therefore, in a natural order of demonstration, take later rank. This will be an advantage to the student, who will, moreover, find the subject more fully explained than it is in Euclid, and in two different points of view. Also, several propositions — the forty-seventh and forty-eighth of Euclid's first book, the thirty-fourth to the thirty-seventh, inclusive, of his third, and the tenth, eleventh, and sixteenth of his fourth—are transferred to the Fifth Chapter as their proper place, they being propositions relating to or depending on squares and rectangles. The forty-seventh, in this treatise, immediately precedes two theorems (Euclid's twelfth and thirteenth, Second Book) from which it should never have been disassociated. It is evident that Euclid placed it in his First Book solely with a view to prove the ninth and tenth theorems of his Second, which, however, could have been easily proved without it. No other motive, at least, can be assigned for his taking it out of its natural place.

The Chapter on Proportion stands in its traditional place—the natural place—at the end of the treatise. The criterion of proportion used is that of Elrington, by *sub*multiples. This test is here adopted because it is more easily understood by young students, and also more conformable to the common notions of proportion. Moreover, it holds good, in all strictness, for commensurable magnitudes; and, as to the incommensurable, it holds equally good if the equisubmultiples taken of the first and third terms be infinitesimals. This question is considered fully in the Sixth and Seventh Notes of the Appendix; where, moreover, an illustration is given of Euclid's criterion which may make its meaning clearly understood by the student. The right conclusion as to the two tests is, probably, that both should be given in a treatise on elementary geometry, each having its own peculiar advantages.

To a few other matters I have to refer. The first is, that, as to strictness of demonstration and the bases on which it is made to rest, there is no geometric principle assumed in this treatise which has not in one form or another been assumed by Euclid. The introduction of Direction as a basis of the science takes the place of Euclid's fifth postulate, the difference being that the reality of direction is far more