AN ELEMENTARY TREATISE ON THE THEORY OF EQUATIONS

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An Elementary Treatise on the Theory of Equations by Samuel Marx Barton

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THEORY OF EQUATIONS

BY

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PREFACE.

In this treatise it is my aim to give the elements of Determinants and the Theory of Equations in a form suitable, both in amount and quality of matter, for use in the undergraduate courses in our colleges and universities. To this end I have endeavored to make the work in every part readily intelligible to the average student who has become proficient in algebra and the elements of trigonometry. All use of the calculus has purposely been avoided. While the presentation of the subject has necessarily been condensed to suit the requirements of college courses, great pains has been taken not to sacrifice clearness to brevity. It is a short treatise, but not a syllabus.

Part I treats of Determinants. The first two chapters give the fundamental theorems, with examples for illustration. The third chapter consists of applications and special forms of determinants, followed by a collection of carefully selected examples. These three chapters on determinants should serve as a helpful introduction to the study of this interesting class of functions.

Part II treats of the Theory of Equations proper. The principal elementary theorems concerning algebraic and numerical equations are deduced. After a brief introduction, giving definitions, etc., there follows a chapter on Complex Quantities, a subject which seems worthy of more space than is usually allotted to it in so elementary a treatise. This chapter, however, is given not so much for use in the chapters that follow, as with the hope that it may prove useful to the

student who pursues later in his course the study of the Theory of Functions. As all the theorems considered have become classic, no special references to authors consulted seem necessary in the body of the book. After Chapter IV I have followed quite closely Burnside and Panton, though in some places the general arrangement has been altered to make the necessary abridgments while securing clearness, and, wherever it seemed desirable, the method of proof has been changed. Almost every theorem is elucidated by the complete solution of one or more representative examples. I desire to call special attention to this feature of the book, which will surely commend itself alike to teacher and pupil. In Chapter XI I have striven to make the rather complicated process of the solution of numerical equations as simple as possible. It would defeat the object of this treatise were much space devoted to these methods, which are laborious and of no great practical value, but what is given is complete in itself. Horner's method is explained in detail.

The following works have been most helpful in the preparation of the treatise, Muir and Burnside and Panton in particular furnishing many examples: Baltzer, *Theorie und Anwendung der Determinanten*, 1881; Burnside and Panton, *Theory of Equations*, 1892; Carnoy, *Cours d'Algèbre Supérieure*, 1892; Hoüel, *Cours de Calcul Infinitésimal*, 1878; Klempt, *Lehrbuch zur Einführung in die Moderne Algebra*, 1880; Muir, *A Treatise on Determinants*, 1882. Todhunter's *Theory of Equations*, Chrystal's *Algebra*, Vol. I, Scott's *Theory of Determinants*, and that excellent little American work by Professor L. G. Weld (*A Short Course in the Theory of Determinants*) should also be mentioned; and the author has consulted with profit the well-known works of Serret, Petersen, Biermann, Matthiessen, and others.

The author gratefully acknowledges his indebtedness to Dr. D. E. Smith, of the State Normal School, at Brockport, N.Y., to Professor William H. Echols, of the University of Virginia, who have read the manuscript and made suggestive criticisms, and to Professor R. D. Bohannan, of the Ohio State University, and Dr. J. H. Gore, of the Columbian University, Washington, who have kindly read the revised proof sheets, and given many valuable suggestions, though he does not wish to hold them in the least responsible for the method followed in the treatment of the subject, nor for any errors that may have crept into the work.

SAMUEL M. BARTON.

SEWANEE, TENN., 1899.

PREFACE TO SECOND EDITION.

IN this Second Edition, a few minor changes have been made in the body of the text, and it is hoped that all typographical errors have been eliminated. The author takes this opportunity of thanking those teachers who have kindly called his attention to errors in the First Edition.

S. M. B.

SEWANEE, 1903.

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