

**THE EARLY MATHEMATICAL
MANUSCRIPTS OF LEIBNIZ; TRANSLATED
FROM THE LATIN TEXTS PUBLISHED BY
CARL IMMANUEL GERHARDT, WITH
CRITICAL AND HISTORICAL NOTES**

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The Early Mathematical Manuscripts of Leibniz; Translated from the Latin Texts Published by Carl Immanuel Gerhardt, with Critical and Historical Notes by J. M. Child

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PREFACE.

A STUDY of the early mathematical work of Leibniz seems to be of importance for at least two reasons. In the first place, Leibniz was certainly not alone among great men in presenting in his early work almost all the important mathematical ideas contained in his mature work. In the second place, the main ideas of his philosophy are to be attributed to his mathematical work, and not *vice versa*. The manuscripts of Leibniz, which have been preserved with such great care in the Royal Library at Hanover, show, perhaps more clearly than his published work, the great importance which Leibniz attached to suitable notation in mathematics and, it may be added, in logic generally. He was, perhaps, the earliest to realize fully and correctly the important influence of a calculus on discovery. The almost mechanical operations which we go through when we are using a calculus enable us to discover facts of mathematics or logic without any of that expenditure of the energy of thought which is so necessary when we are dealing with a department of knowledge that has not yet been reduced to the domain of operation of a calculus. There is a frivolous objection raised by philosophers of a superficial type, to the effect that such economy of thought is an attempt to substitute unthinking mechanism for living thought. This contention fails of its purpose through the simple fact that this economy is only used in certain circumstances. In no science do we try to make subject to a mechanical calculus any trains of reasoning except such that have not been the object of careful thought many times previously. Not only so, but this reasoning has been universally recognized as valid, and we do not wish to waste energy of thought in repeating it when so much remains to be discovered by means of this energy. Since the time of Leibniz, this truth has been recognized, explicitly or implicitly, by all the greatest mathematical analysts.

It is not difficult to connect with this great idea of the importance of a calculus in assisting deduction the many unfinished plans of Leibniz; for instance, his projects for an encyclopædia of all science, of a general science, of a calculus of logic, and so on. These projects, however, do not come within the field of this essay, which is a collection of various articles which appeared in *The Monist* from 1916 to 1918; our concern will be the various influences on Leibniz in his earliest original mathematical work. Merely biographical details do not seem to be relevant.

In writing the following pages, I have been greatly influenced and helped by the emphasis laid by Mr. Philip E. B. Jourdain upon the importance which Leibniz himself attached to the notion of a calculus in general, and his own operational calculus in particular; he it was who also suggested that I should undertake a critical translation of the early mathematical manuscripts of Leibniz; to him also I am greatly indebted for many points upon which I was unable to make up my mind on the evidence that I could get from the manuscripts alone. I have also to thank Mr. W. J. Greenstreet for looking through my articles before they were assembled for the purpose of this volume, and for making some valuable suggestions. My excuse for publishing these manuscripts, enlarged with so many and such long critical notes, must lie in the fact that I have made a careful study of the work of Barrow, and have recognized, perhaps at more than its true value, though I do not think so personally, its great genius and the influence it had on Leibniz. The opportunities it was capable of affording to Leibniz, the greater likeness that the work of Leibniz bears to that of Barrow than to that of Newton, have forced me to the conclusion that Leibniz was in no way indebted to Newton for anything, yet his statement in a letter to the Marquis d'Hospital, that he was under no obligation to Barrow *for his methods*, is absolutely correct.

J. M. CHILD.

DERBY, ENGLAND, September, 1919.

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial reporting and auditing.

2. The second part of the document outlines the various methods and techniques used to collect and analyze data. This includes both qualitative and quantitative approaches, as well as the use of statistical tools and software to process large volumes of information.

3. The third part of the document focuses on the interpretation and presentation of the collected data. It discusses how to identify trends, patterns, and anomalies, and how to communicate these findings effectively to stakeholders through reports, presentations, and visual aids.

4. The fourth part of the document addresses the ethical considerations and challenges associated with data collection and analysis. It highlights the need for privacy, confidentiality, and integrity, and provides guidance on how to navigate these issues responsibly.

5. The fifth part of the document concludes by summarizing the key points and offering final thoughts on the importance of data-driven decision-making in today's business environment. It encourages ongoing learning and adaptation to new technologies and methodologies.

INTRODUCTION.

A PART from the intrinsic interest which the autograph writings, and more particularly the earlier efforts, of any of the prime movers in any branch of learning possess for the historical student, there is a special interest attached to the manuscripts and correspondence of Leibniz. They are invaluable as an aid to the study of the part that their author played in the invention and development of the infinitesimal calculus. More especially is this so in the case of Leibniz; for the matter, upon which this essay is founded, unearthed by Dr. C. I. Gerhardt in a mass of papers belonging to Leibniz that had been preserved in the Royal Library of Hanover, contained holographs previously unpublished.

The most important of these, for our purpose, were edited, with full notes and a commentary, by Gerhardt, in three separate volumes, under the respective titles:

1. *Historia et Origo Calculi Differentialis, a G. G. Leibnizio conscripta.* Hanover, 1846.
2. *Die Entdeckung der Differentialrechnung durch Leibniz.* Halle, 1848.
3. *Die Geschichte der höheren Analysis; erste Abtheilung, Die Entdeckung der höheren Analysis.* Halle, 1855.*

* For abbreviations used in this volume for these and other works, see the Bibliography given at the end.

The present time,¹ the two-hundredth anniversary of the death of Leibniz, would seem to be a most suitable one for publishing an English translation of these manuscripts.

For the present purpose, it will be convenient to group the manuscripts in two sections, of which the first will consist of Leibniz's own account of his work. Under the heading § 1, (p. 11), is given a fairly literal translation of a postscript from Leibniz to Jakob (i. e., James) Bernoulli, "which was written from Berlin in April 1703, and then cancelled and a postscript on a totally different subject substituted."² This is a communication to a more or less intimate friend. It is therefore naturally not such a considered composition as the second account that Leibniz gives of his work in the *Historia* mentioned above, of which a full translation is given below under the heading § 2. It is important to bear this point in mind when comparing the two accounts together, for any slight discrepancies that may be noticed are, feasibly at least, to be accounted for by the different circumstances of the compositions. The latter account bears the impress of being fairly fully revised and made ready for press, and the facts marshalled to make an impressive or, as some would have it, plausible whole; it was probably finished just before the death of Leibniz, and represents his answer to the *Commercium Epistolicum* of unsavory memory. The death of Leibniz in November 1716 was probably the cause which prevented its publication, or at least the chief reason.

It is not my intention to enter into a discussion about the *Commercium Epistolicum*; this has probably had the last word said upon it that it is possible to say with the help of the existing authentic material that is possessed by the present-day historians of mathematics. Further,

¹ This appeared in *The Monist* for October, 1916.

² G. 1848, p. 29; see also G. *math.*, III, pp. 71, 72, and Cantor, III, p. 40.

if I hold quite other views as to the possible source of Leibniz's inspiration, if indeed he is not to be credited with perfectly independent discovery. I will therefore, as far as I may, refrain from allusion to the *Commercium Epistolicum*, except to second the plea of its perfectly disgraceful unfairness, as made by Leibniz.² I have suggested above that the *Historia* was intended by Leibniz as a statement of his side of the case, and as an answer to the attack made upon him. This account of his work, although written in the third person, "by a friend who knew all about the matter,"³ is, on the authority of Gerhardt, undoubtedly by Leibniz himself. Without in any way impugning this authority, I cannot help thinking it would have been more satisfactory if I could have included herein photographic copies of parts of this manuscript; but this is impossible at the time of writing.

The reasons for the delay in the preparation of the *Historia* are stated in the manuscript itself; and later I shall have occasion to discuss these. In order that the remarks made may in all cases be perfectly intelligible, I must here give a very short account⁴ of the history of the

² A fair-minded consideration, like everything emanating from the pen of De Morgan, is given of the matter in a recent edition of his *Essays on the Life and Work of Newton*. The tale is told with the charm characteristic of De Morgan, and the edition is rendered very valuable by the addition of notes, commentary, and a large number of references supplied by the editor, P. E. B. Jourdain (Open Court Publishing Co.). Special attention is directed to De Morgan's summary of the unfairness of the case in Note 3 at the foot of pages 27-28.

³ See under 11 below: also cf. the original Latin as given in G. 1846, p. 4, "per amicum conscium."

⁴ The account here given is substantially that given by Gerhardt in an article in Grunert's *Archiv der Mathematik und Physik*, 1856; pp. 125-132.

This article is written in contradiction to the view taken by Weissenborn in his *Principien der höheren Analysis*, Halle, 1856. It is worthy of remark that the partisanship of Gerhardt makes him omit in this article all mention of the review which Leibniz wrote for the *Acta Eruditorum* on Newton's work, *De Quadratura Curvarum*, which really drew upon him the renewal of the attack, by Keill. The passage which was objected to by the English mathematicians as being tantamount to a charge of plagiarism, in addition to the insult implied, according to their thinking, in making Newton the fourth proportional to Cavalieri, Fabri and Leibniz, is however given by Gerhardt in his preface to the *Historia* (G. 1846, p. vii).